



IEEE Ultrasonics Symposium 2009
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Short Course 6A: Estimation and Imaging of Blood Flow Velocity

Hans Torp and Lasse Løvstakken

Department of Circulation and Medical Imaging
Norwegian University of Science and Technology
Trondheim, Norway

Web-page: <http://folk.ntnu.no/lovstakk/dopplershortcourse2009>
(contains supporting literature and Matlab demonstrations)

E-mail: Hans.Torp(at)ntnu.no, Lasse.Lovstakken(at)ntnu.no

Course summary: This course provides a basic understanding of the physical principles and signal processing methods for estimation of blood flow velocity. The course begins with an overview of currently used techniques for velocity estimation using pulsed- and continuous-wave Doppler, and color flow imaging. Fundamental challenges related to data acquisition will be presented. Further, statistical models for the received signal as well as commonly used velocity estimators will be developed. The suppression of clutter from slowly moving targets is central to all processing schemes and will be given special attention. Finally, an introduction to advanced topics such as adaptive clutter filtering and 2-D / 3-D vector velocity estimation techniques will be given. Principles and practical limitations will be discussed, and potential clinical applications will be shown.

Course outline

Lecture 1 – Basic principles of Doppler ultrasound

- General introduction to course
- Clinical motivation for velocity measurements with ultrasound and a short Doppler history lesson
- CW-Doppler measurements
- Random signal model and spectrum analysis
- PW-Doppler measurements

Lecture 2 – Color flow imaging

- Clinical motivation and short history lesson
- Data acquisition in CFI
- Mean (axial) velocity estimation in CFI
- Clutter filtering in CFI
- Display techniques in CFI

Lecture 3 – Advanced topics, trade-offs and limitations

- Limitations of conventional Doppler / CFI techniques
- Patient safety aspects
- Advanced axial velocity estimators
- Adaptive clutter filtering

Lecture 4 – Future directions

- 2D vector velocity estimation
- High frame rate CFI
- Real-time 3-D CFI

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Doppler Short Course '09

Estimation and imaging of blood flow velocity

Part I: Basic principles

2009 Ultrasonics Symposium, Rome, Italy

Hans Torp and Lasse Løvstakken

Department of Circulation and Medical Imaging

Norwegian University of Science and Technology



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Why measure and visualize blood flow?

- Measure blood supply to an organ
- Angiography without contrast agent
- Measure pressure drop through an stenotic valve
-



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Course outline

- Lecture 1: Basic Doppler techniques (Torp)
 - Basic principles of Doppler ultrasound
 - PW- and CW-Doppler
- Lecture 2: Color flow imaging (Løvstakken)
 - Data acquisition
 - Velocity estimation
 - Clutter filtering
 - Display techniques
- Lecture 3: Advanced topics (Løvstakken/Torp)
 - Limitations of conventional techniques
 - Patient safety issues
 - Advanced velocity estimators
 - Adaptive clutter filtering
- Lecture 4: Future directions (Løvstakken)
 - Vector-velocity estimation
 - High frame rate imaging
 - Real-time 3-D CFI

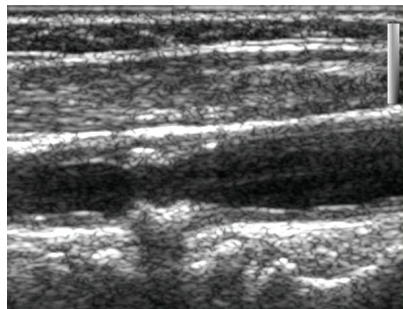
Course objectives

- Review and understanding of basic principles of ultrasound Doppler imaging
- Emphasis on signal models and -processing
- Clinical examples in cardiology and vascular imaging

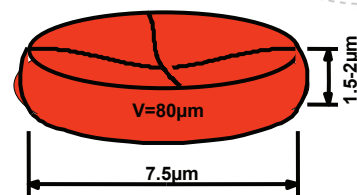
Lecture 1 outline

- Principles and applications of continuous-wave (CW) Doppler
- Random signal model and spectrum analysis
- Pulsed-wave (PW) Doppler
- Time shift versus Doppler shift

Red blood cells are hardly visible in the ultrasound image



Carotid artery with calcified plaque



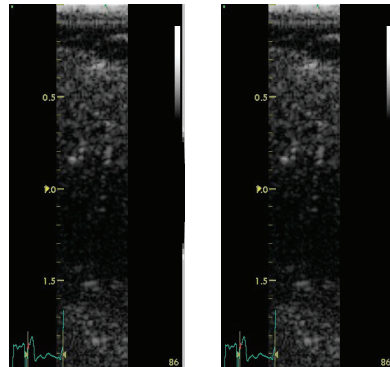
Red blood cell

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High frequency / low acoustic noise → Blood visible

13 MHz epicardial probe, left ventricle
450 frames/sec

High frame-rate required to see the motion



Real-time

Slow motion 10%

Blood Velocity	Displacement 50 frames/sec	Minimum Frame rate
5 cm/sec	1 mm	50 fr/sec
50 cm/sec	1 cm	500 fr/sec
5.0 m/sec	10 cm	5000 fr/sec

For high velocity jet flow,
e.g. in heart valve diseases
Frame rate ~ 5000 is required
to observe and measure velocity

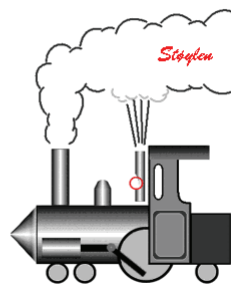
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The Doppler effect



Christian Andreas Doppler
(1803 - 1853)
Described the Doppler
effect to *light waves*

Cristoph Ballot
demonstrated the Doppler effect for *sound waves* (1845)

Doppler shift:

$$f_d = f_o v/c$$

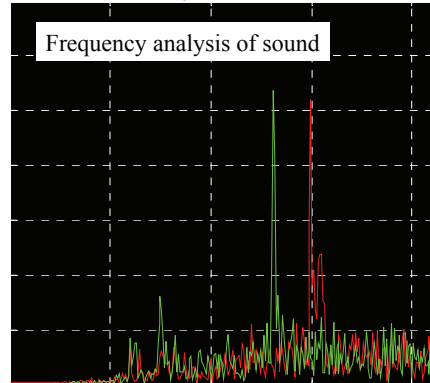
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Doppler speed control



Dopplershift: $\pm 9.55 \text{ Hz} \sim \pm 6.8 \%$

Motor bike speed: $340[\text{m/s}] * 6.8/100 * 3.6 = 83.3 \text{ km/h}$

Speed limit = 80 km/h

Frequency before pass by: 130.85 Hz

Frequency after pass by: 150.80 Hz

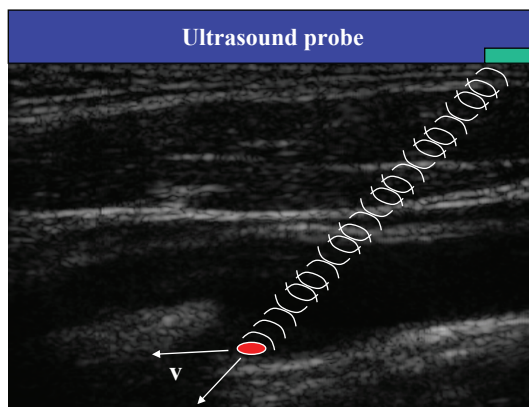
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Doppler shift from moving blood



Dopplershift

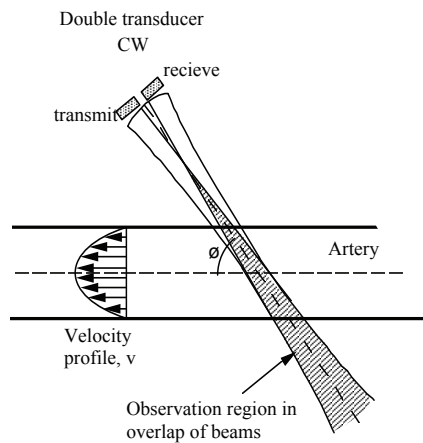
$$\begin{aligned} f_d &= f_o v/c \\ &+ f_o v/c \\ &= 2 f_o v/c \end{aligned}$$

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Continuous wave Doppler



Blood velocity calculated from measured Doppler-shift

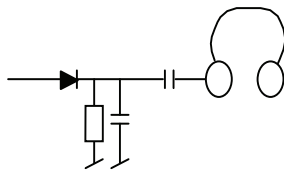
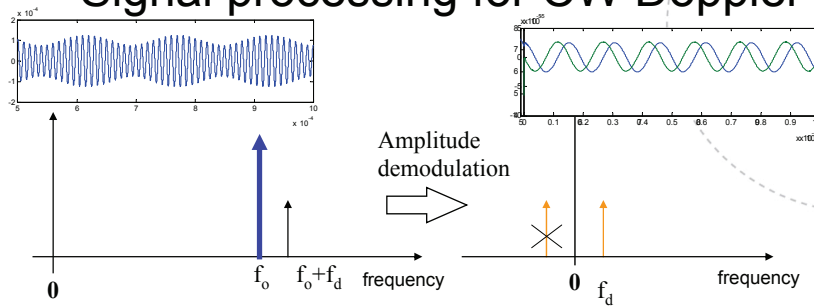
$$f_d = \frac{2f_0 v \cos(\theta)}{c}$$

$$v = \frac{c \cdot f_d}{2f_0 \cos(\theta)}$$

f_d :	Dopplershift
f_0 :	transmitted frequency
v :	blood velocity
θ :	beam angle
c :	speed of sound in blood (1540 m/s)

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Signal processing for CW Doppler



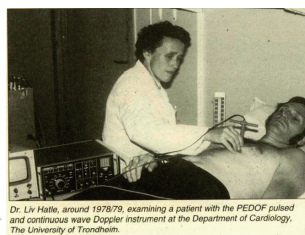
Amplitude demodulation

Satsumora, Japan, 1957

Complex Demodulation
Separate positive and negative
Doppler shift

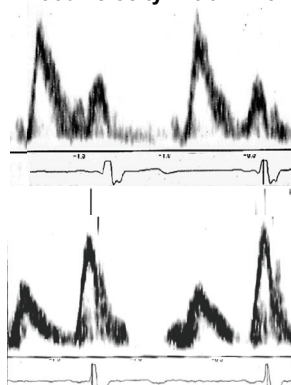
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Doppler blood flow meter (Pedof 1976)



Dr. Liv Halle, around 1978/79, examining a patient with the PEDOF pulsed and continuous wave Doppler instrument at the Department of Cardiology, The University of Trondheim.

Blood velocity Mitral inflow

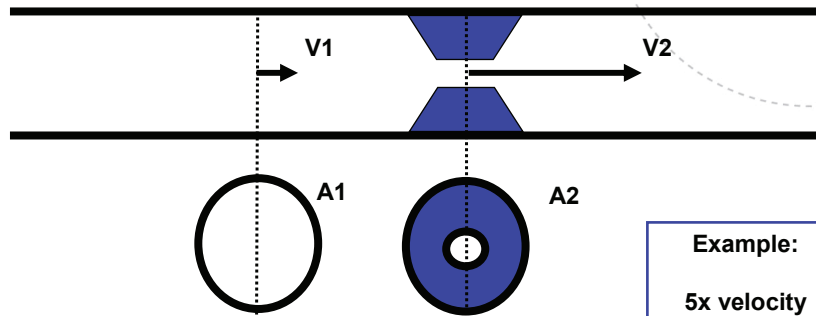


Normal relaxation

Delayed relaxation

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Continuity of flow to assess stenosis



$$V1 * A1 = V2 * A2$$

$$\% \text{ reduction} = \frac{A1 - A2}{A1} = \frac{V2 - V1}{V2}$$

Example:

**5x velocity
corresponds to
80% stenosis**

Area reduction depends only on the velocity $V2/V1$;
independent of diameter and angle

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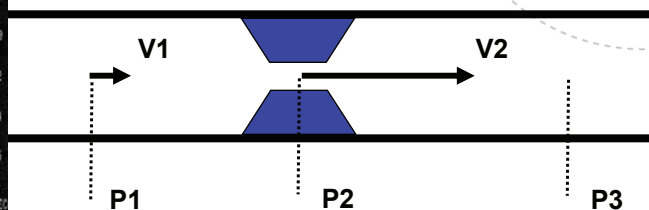
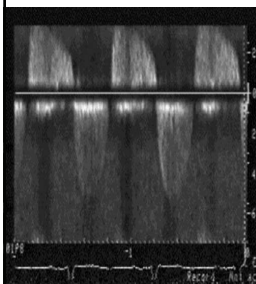
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Reduction in blood pressure in a stenotic artery

Bernouli's equation describes pressure drop due to increase in kinetic energy



Pressure drop (gradient) : $P1 - P3 = 4 V2^2$

Example: 80% aortic-stenosis

$V1 = 1 \text{ m/s}$ $V2 = 5 \text{ m/s}$ pressure-gradient $4 * 5 * 5 = 100 \text{ mmHg}$

Normal aortic pressure 120 mmHg corresponds to
220 mmHg ventricular pressure!

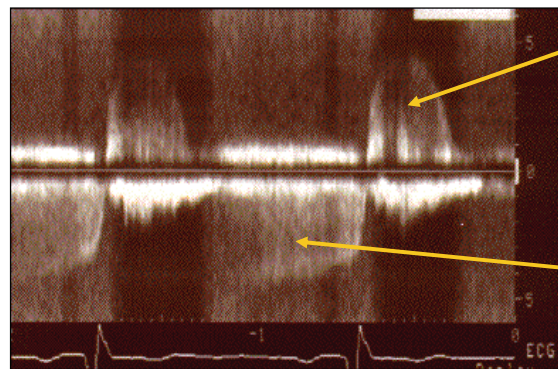
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Aortic stenosis and regurgitation by CW Doppler



Stenosis
(forward flow)

Regurgitation
(reverse flow)

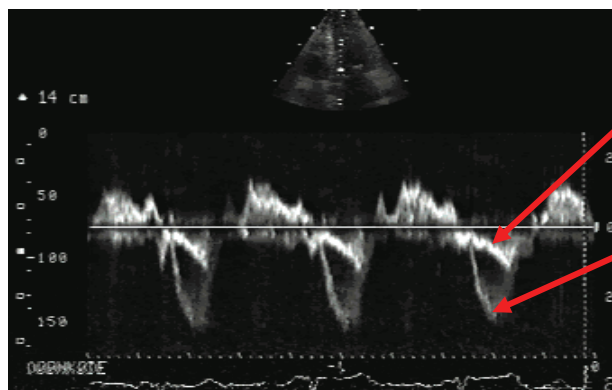
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Doppler spectrum analysis: Velocities separated in frequency



LV blood flow

Aortic blood flow

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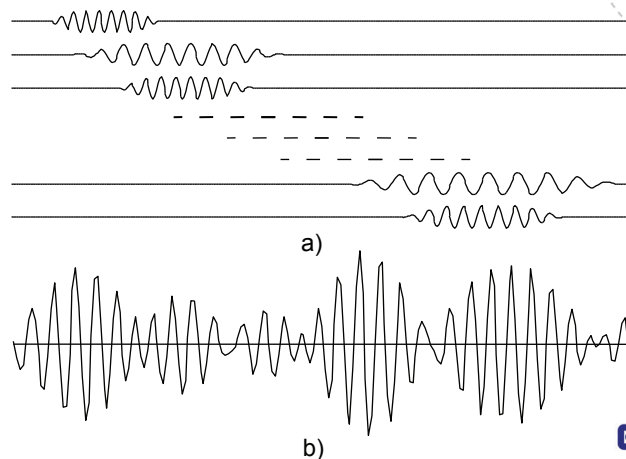
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Lecture 1 outline

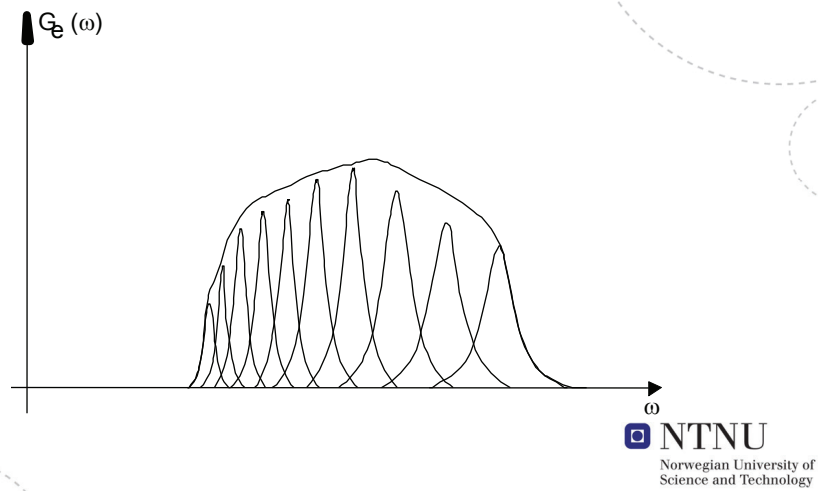
- Principles and applications of CW Doppler
- **Random signal model and spectrum analysis**
- Pulsed wave Doppler
- Time shift versus Doppler shift

Signal from a large number of red blood cells
add up to a Gaussian random process



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Power spectrum of the Doppler signal represents the distribution of velocities within the blood vessel

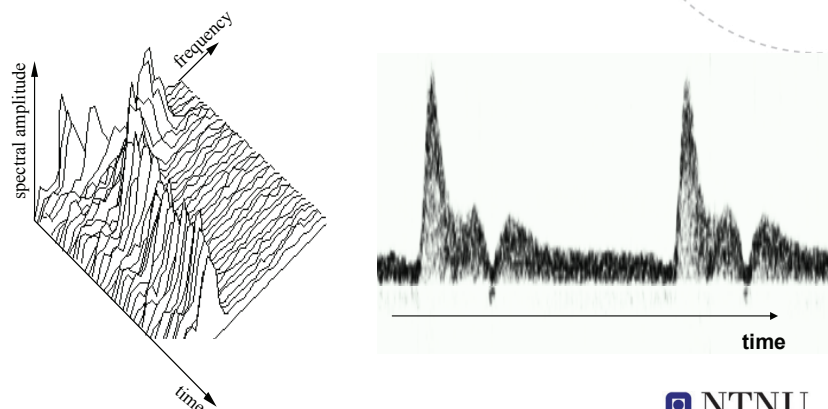


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Doppler spectrum



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Definition of Complex Gaussian process

$$p(z) = \pi^{-N} |\mu|^{-1} e^{-z^T \mu^{-1} z}$$

signal vector $z = z(1), z(2), \dots, z(N)$

Covariance matrix $\mu = \langle z^T z \rangle = \langle z(k)^* \cdot z(n) \rangle_{k,n}$

$\langle - \rangle$ is expected value (ensemble average)

Note that: $\langle z(k) \cdot z(n) \rangle = 0$

Stationary complex Gaussian process

Autocorrelation function

$$R(m) \equiv \langle z(n)^* z(n+m) \rangle \quad m = 0, \pm 1, \pm 2, \dots$$

Power spectrum

$$G(\omega) \equiv \sum_m R(m) e^{-i\omega m} \quad ; \quad -\pi < \omega < \pi$$

Autocorrelation function = coefficients in Fourier series of G

$$R(m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\omega G(\omega) e^{i\omega m}$$

Power spectrum estimate

Statistical properties

Power spectrum estimate:

$$G_N(\omega) = \frac{1}{N} |Z_N(\omega)|^2 \quad Z_N(\omega) = \sum_{m=-\infty}^{\infty} w_N(m) z(m) e^{-i\omega m}$$

Expected value:

$$\langle G_N(\omega) \rangle = \frac{1}{2\pi} \int_{-\pi}^{\pi} d\lambda |W(\lambda)|^2 G(\omega - \lambda) \quad W(\omega) = \sum_m w_N(m) e^{-i\omega m}$$

Power spectrum estimate

Statistical properties

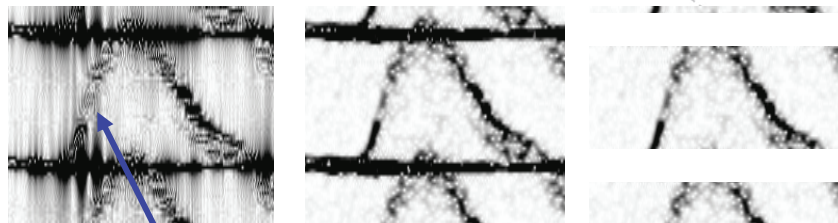
Covariance:

$$\text{cov}(G_N(\omega), G_N(\omega + \Delta)) = \left| \frac{1}{2\pi N} \int_{-\pi}^{\pi} d\lambda W(\lambda) W^*(\lambda - \Delta) G(\omega - \lambda - \Delta) \right|^2$$

$$= \begin{cases} \langle G_N(\omega) \rangle^2 & \text{when } \Delta = 0 \\ 0 & \text{when } |\Delta| > 1/N \end{cases}$$

Exponential probability density distribution
Fractional variance = 100%

Clutter noise in spectral Doppler



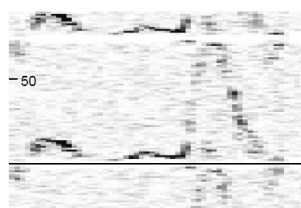
Rectangular window

Hamming window

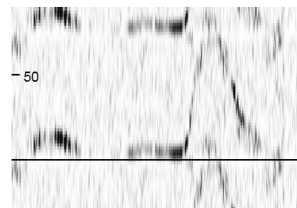
High pass filter

Interfering sidelobes from clutter signal

Frequency resolution versus temporal resolution



Window length 64



Window length 16

Time-bandwidth product: $\Delta f_d * \Delta T = 1$

Properties of power spectrum estimate

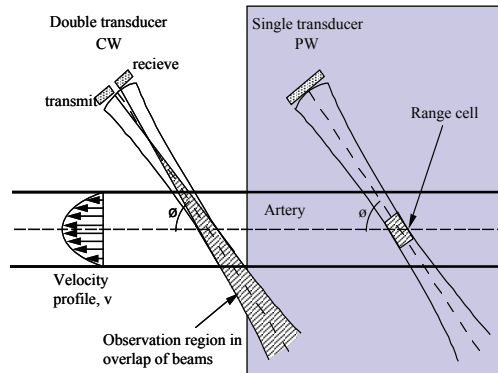
- Fractional variance = 1 independent of the window form and size
- $G_N(\omega_1)$ and $G_N(\omega_2)$ are uncorrelated when $|\omega_1 - \omega_2| > 1/N$
- Increasing window length N gives better frequency resolution, but no decrease in variance
- Smooth window functions give lower side lobe level, but wider main lobe than the rectangular window
- Decrease in variance can be obtained by averaging spectral estimates from different data segments.

Lecture 1 outline

- Principles and applications of CW Doppler
- Random signal model and spectrum analysis
- **Pulsed wave Doppler**
- Time shift versus Doppler shift

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Continuous Wave Doppler → Pulsed Wave Doppler



Sample volume length
 $= cT/2$

T = pulse length

Signal from all scatterers
within the ultrasound beam

Signal from a limited
sample volume

Satamura, Japan, 1957

Perroneau, Baker, Wells 1967-69

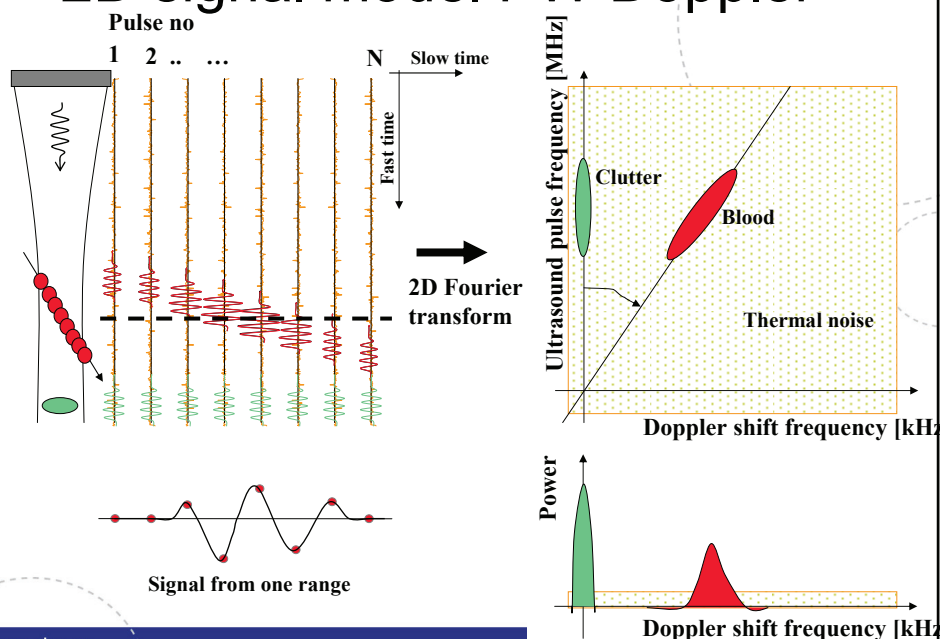
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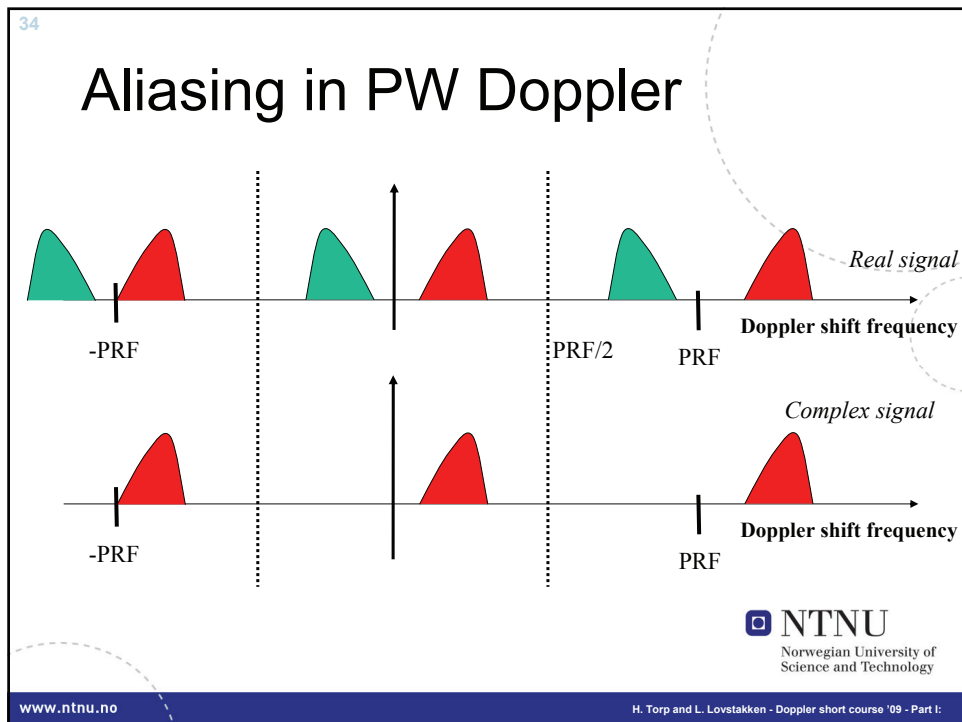
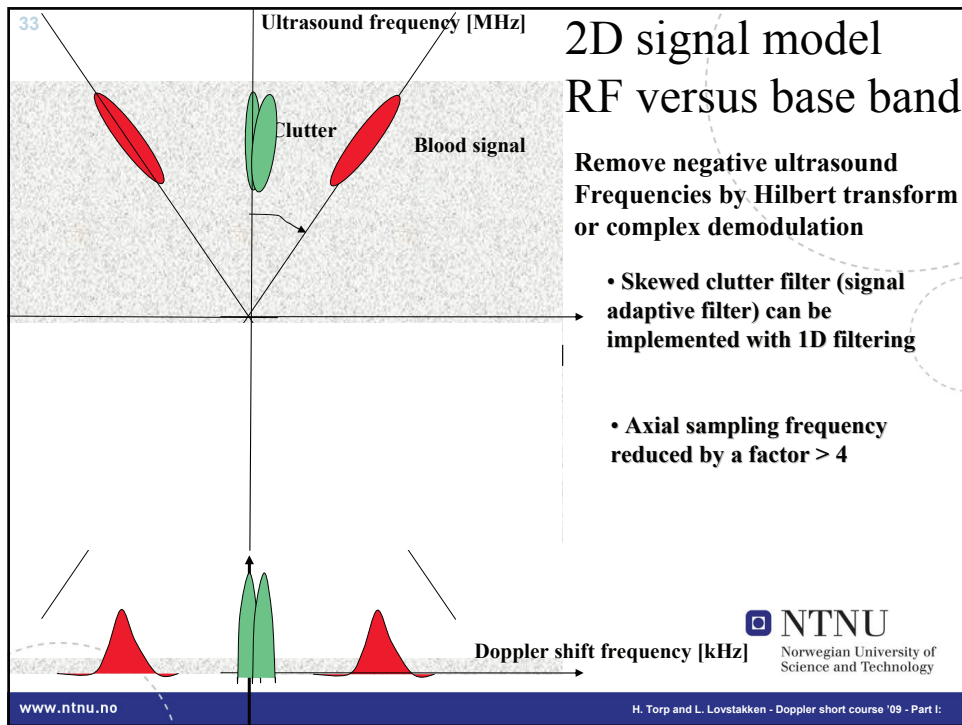
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2D signal model PW Doppler



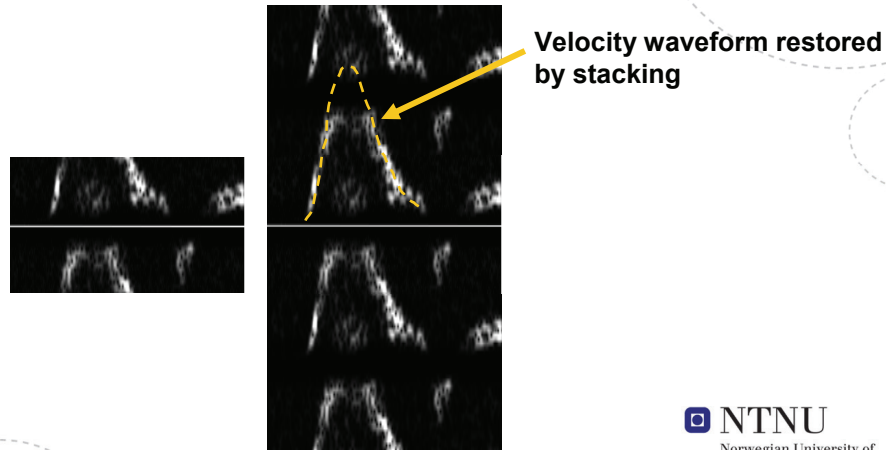
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Aliasing in PW Doppler

Example: Subclavian Artery



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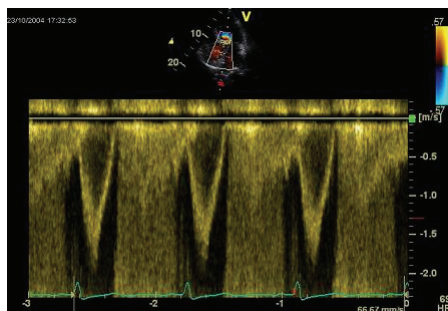
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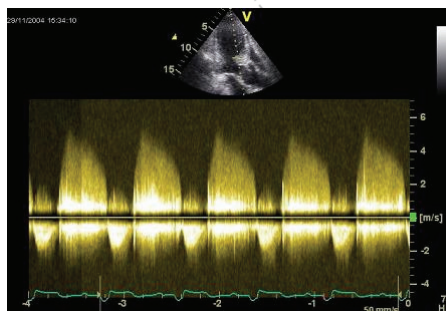
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Aliasing PW versus CW Doppler

Example: Aortic valve leakage jet



PW Doppler



CW Doppler

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Summary PW Doppler

- Complex demodulation give direction information of blood flow
- PW Doppler suffers from aliasing in many cardiac applications
- Aliasing can be restored by baseline shift, or stacking, if bandwidth is limited

Lecture 1 outline

- Principles and applications of CW Doppler
- Random signal model and spectrum analysis
- Pulsed wave Doppler
- **Time shift versus Doppler shift**

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Is Pulsed wave Doppler based on Doppler shift or time shift?

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Ultrasound's interaction with blood

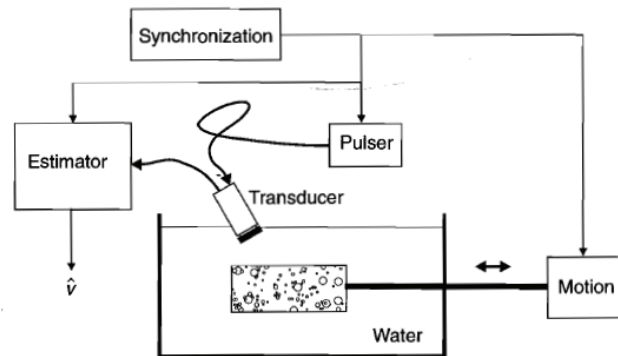


Fig. 4.4. Basic setup to illustrate Doppler effect and time shift effects.

J.A.Jensen: Estimation of blood velocities using Ultrasound

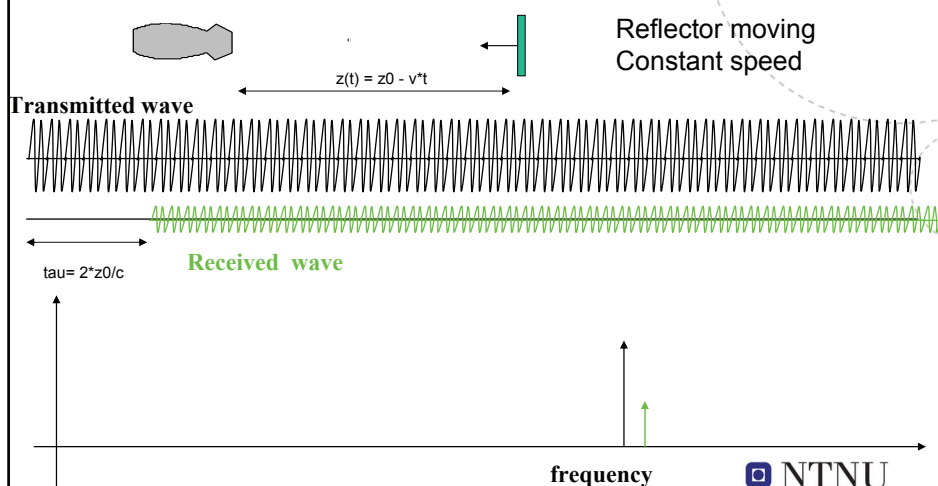
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Continuous wave Doppler Doppler shift or time shift?



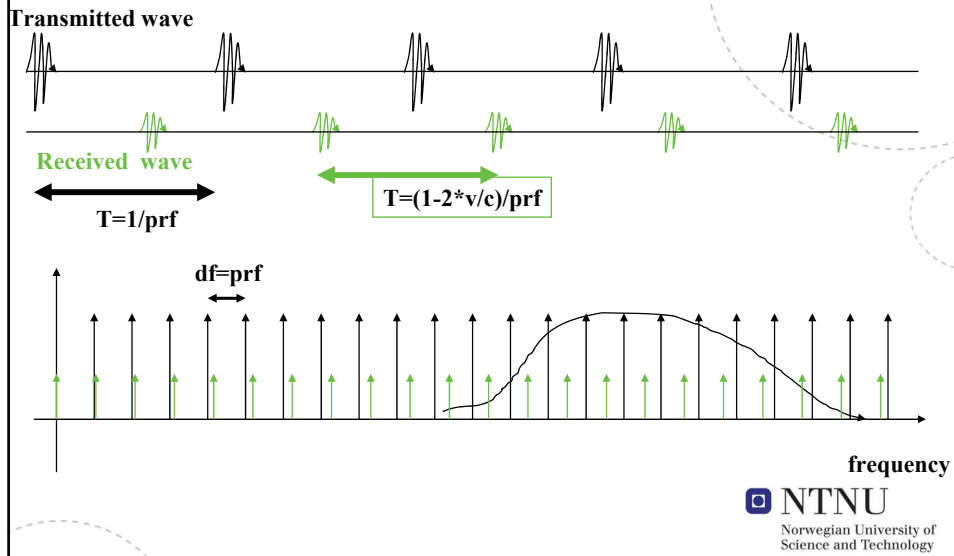
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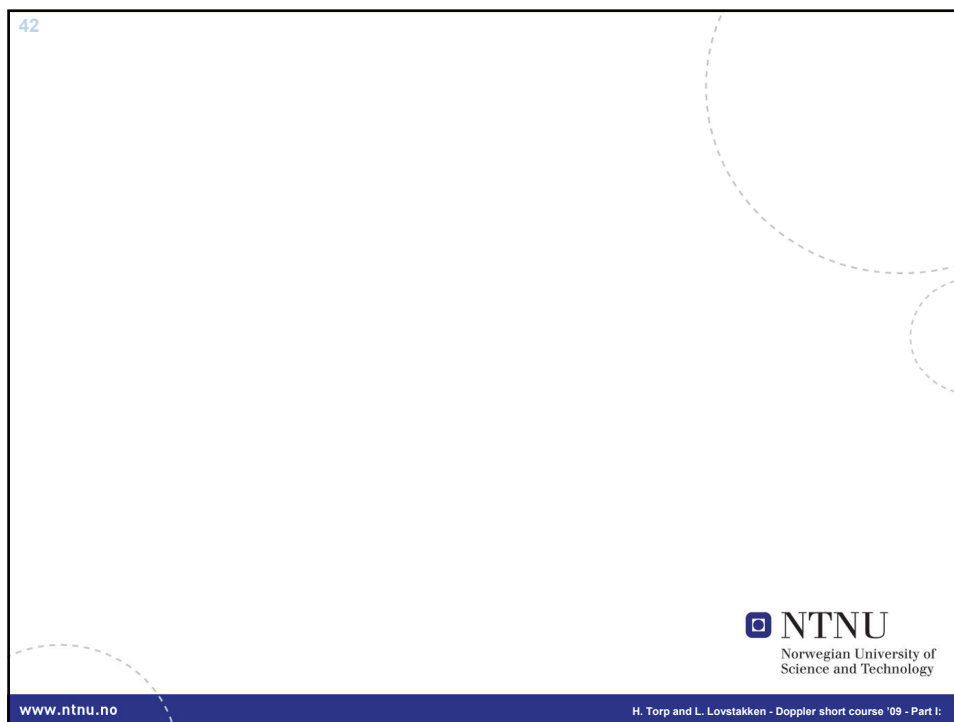
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Pulsed wave Doppler Doppler shift or time shift?



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Estimation and imaging blood flow velocity

Part II: Color Flow Imaging

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Department of Circulation and Medical Imaging

Norwegian University of Science and Technology



Lecture outline

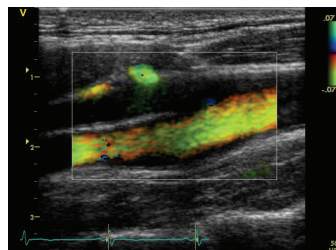
- General introduction to color flow imaging
 - Examples of clinical use, and a brief history lesson
- Data acquisition strategies
 - Acquiring the temporal sequence of samples
- Velocity estimation techniques
 - Estimation of Doppler signal parameters
- Clutter filtering
 - Attenuation of interfering signal from stationary tissue
- Display strategies in CFI



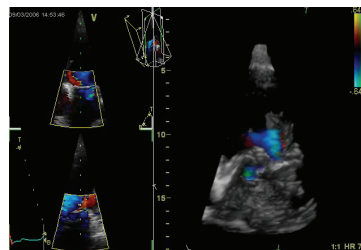
General introduction to CFI

Color flow imaging

- Displays a *color coded map* of the *axial* blood velocity in a 2-D or 3-D region of interest
- Used in a wide range of diagnostic contexts in today's hospitals



2-D CFI of a carotid bifurcation

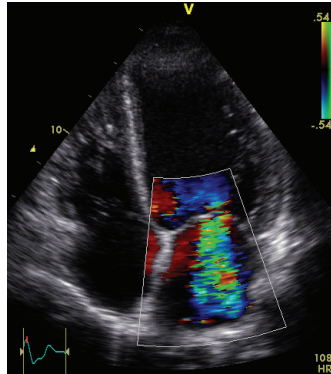


3-D CFI of a mitral regurgitation jet

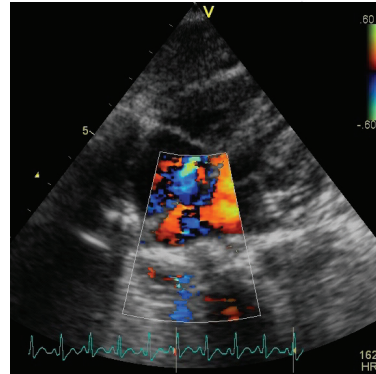
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Clinical examples

Mitral valve regurgitation jet



Atrial septum defect shunt flow



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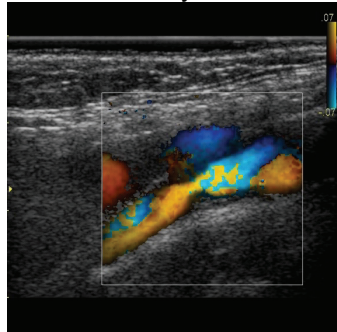
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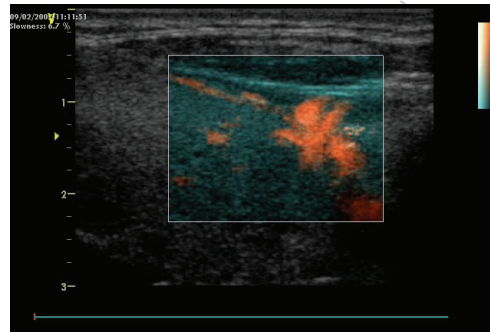
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More clinical examples

Carotid artery stenosis



Thyroid nodule vascularization



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A brief history lesson

- Technology and research progressed from single-range gate to multi-range gated Doppler, and further to 2-D Doppler imaging

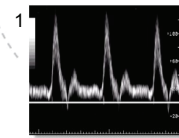
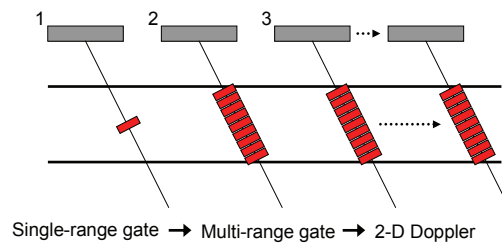
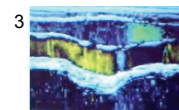
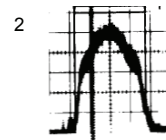


Figure 2



2-D color flow image of the carotid and jugular vein (top) from the Brandestini group in 1975

Image source: J. Woo, A short History of the development of Ultrasound in Obstetrics and Gynecology, <http://www.ob-ultrasound.net/history.html>

A brief history lesson

- Real-time CFI was first commercially available in the mid-eighties
 - Aloka (1985, Japan), Toshiba (1985, Japan), Quantum (1986, US), Vingmed (1986, Norway)

Aloka SSD-880CW



Quantum QAD-1



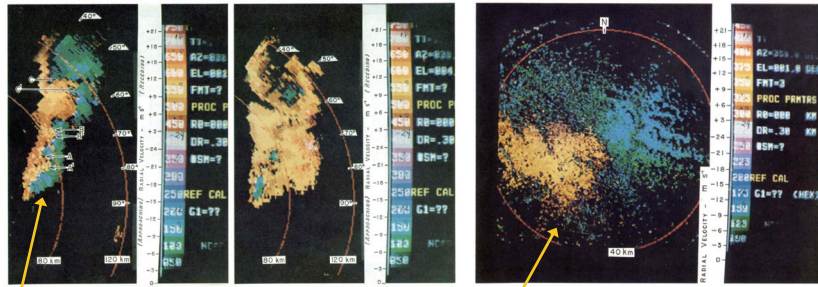
Vingmed CFM-700



Source: J. Woo, A short History of the development of Ultrasound in Obstetrics and Gynecology, <http://www.ob-ultrasound.net/history.html>

A brief history lesson

- However, real-time processing and display of color-Doppler images in weather RADAR was available ten years earlier (!)



Storm signs

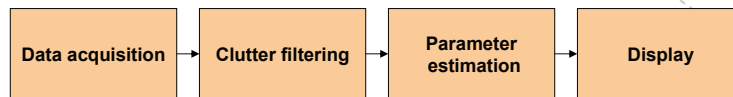
Insect signs

Source: G. R. Gray et al, Real-time color-Doppler RADAR display, *Bulletin of the American Meteorological Society*, vol. 56(6), 1975

CFI acquisition and processing

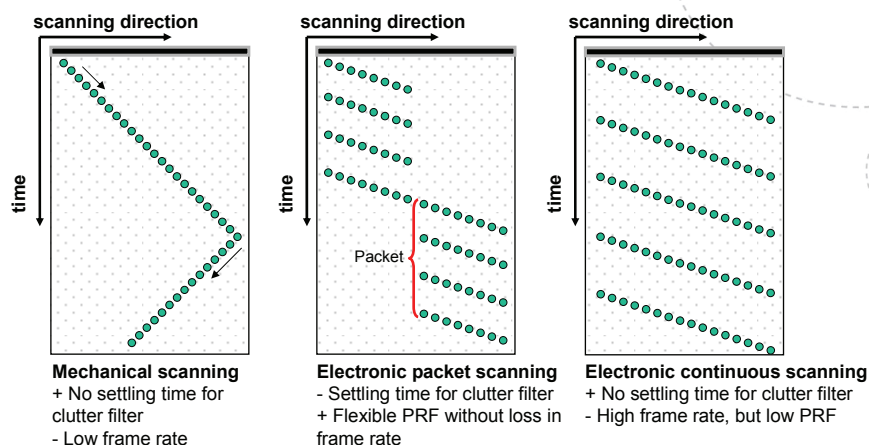
CFI processing chain

Processing chain of conventional CFI



- CFI data acquisition
 - Scanning operation and pulse sequence
- Clutter filtering (wall filtering)
 - Attenuating interfering signal from stationary tissue
- Doppler parameter estimation
 - Estimation of Doppler power, mean-frequency, and bandwidth
- Display
 - Color encoding of Doppler parameters

Data acquisition in CFI



Interleaved packet acquisition



- Overall frame rate increased by the interleave factor
- Maximum PRF limited by image depth, multiple reflections, hardware
- Especially important when imaging low-velocity flow, i.e. with a low user chosen PRF such as peripheral vascular imaging

Reference: R. H. Chesarek, *Ultrasound imaging system for relatively low-velocity blood flow at relatively high frame rates*. USA, Patent no. 4888694, Quantum Medical System, Inc., 1989.

Packet / ensemble size in CFI

- Increasing the packet size in CFI will:
 - + Lead to more efficient clutter filtering
 - + Lower the variance of the Doppler parameters
 - Reduce the overall frame rate dramatically
 - May lead to visible artificial lags of the flow field in the image from one side of the image to the other
- How many samples are necessary?
 - Application dependent, typically 8-16 samples
 - Cardiac imaging (deep, high dynamics): packet size = 8-10
 - Vascular imaging (shallow, lower dynamics): packet size = 10-16
 - Abdominal imaging (deep, lower dynamics): packet size = 10-12

Signal-to-noise considerations

Signal-to-noise is one of the most crucial design criteria in Doppler acquisition

- Similar as for PW-Doppler: total SNR $\sim 1/B$, spectral SNR $\sim 1/B^2$
- The received signal from blood is proportional to the length of the transmitted pulse (incoherent sum of burstlets)
 - Increase the number of pulse cycles
 - Decrease the pulse center frequency
- The optimal receive filter is approximatively given by a rectangular filter with length equal to the emitted pulse
 - Boxcar integrator over the pulse length
- Optimize TGC to achieve a constant noise floor throughout depth

Reference: K. Kristoffersen, *Optimal receiver filtering in pulsed Doppler ultrasound blood velocity measurements*, IEEE Trans. Ultrason., Ferroelec., and Freq. Contr., vol. 33(1), pp. 51-58, Jan. 1986

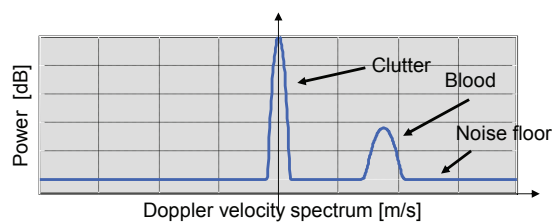
CFI velocity estimation

Doppler signal model

- The Doppler spectrum may consists of three components, clutter c , blood b , and thermal noise n

$$\mathbf{x} = \mathbf{c} + \mathbf{n} + \mathbf{b} \quad \mathbf{x} = [x(1), \dots, x(N)]^T$$

- Typical clutter/signal level: 20 – 80 dB
- Signal from blood is characterized by a complex Gaussian process



Doppler parameter estimation

Doppler parameter estimation in CFI has focused on the first three moments of the Doppler spectrum, which equals the mean power, mean frequency, and bandwidth (rms):

$$P = \int_{-\infty}^{\infty} G(\omega) d\omega \quad \bar{\omega} = \frac{\int_{-\infty}^{\infty} \omega \cdot G(\omega) d\omega}{\int_{-\infty}^{\infty} G(\omega) d\omega} \quad B_{\text{rms}}^2 = \frac{\int_{-\infty}^{\infty} (\omega - \bar{\omega})^2 \cdot G(\omega) d\omega}{\int_{-\infty}^{\infty} G(\omega) d\omega}$$

However: estimating the Doppler power spectrum and integrating is not a practical solution.

Time (phase) domain approaches has several qualities

- Less computationally expensive
- Robust in low signal-to-noise ratios
- Velocity range covering the full Nyquist spectrum width

Time domain formulation

The **Wiener-Khinchin** formula relates the autocorrelation function and the power spectral density function:

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} G(\omega) e^{j\omega\tau} d\omega$$

Derivatives with respect to tau gives:

$$\dot{R}(\tau) = \frac{j}{2\pi} \int_{-\infty}^{\infty} \omega G(\omega) e^{j\omega\tau} d\omega \quad \ddot{R}(\tau) = \frac{-1}{2\pi} \int_{-\infty}^{\infty} \omega^2 G(\omega) e^{j\omega\tau} d\omega$$

Yields time-domain expressions for power, mean frequency, and bandwidth (rms):

$$P = R(0) \quad \bar{\omega} = -j \frac{\dot{R}(0)}{R(0)} \quad B^2 = \left[\frac{\dot{R}(0)}{R(0)} \right]^2 - \frac{\ddot{R}(0)}{R(0)}$$

The autocorrelation method

However: Accurate estimates of the derivatives of the autocorrelation function can be difficult to achieve. Therefore an alternative formulation is used:

Correlation function in polar form: $R(\tau) = A(\tau) \exp[j\phi(\tau)]$

Yields the following mean frequency and bandwidth estimate:

$$\bar{\omega} = -j \frac{\dot{R}(0)}{R(0)} = \dot{\phi}(0) \quad \bar{\omega} \cong \frac{\phi(T_{PRF}) - \phi(0)}{T_{PRF}} = \frac{1}{T_{PRF}} \arg[R(T_{PRF})]$$

$$B^2 = -\frac{\ddot{A}(0)}{A(0)} \approx \frac{2}{T_{PRF}^2} \left[1 - \frac{A(T_{PRF})}{A(0)} \right] = \frac{2}{T_{PRF}^2} \left[1 - \frac{|R(T_{PRF})|}{R(0)} \right]$$

In other words: The power, mean frequency and bandwidth of the Doppler spectrum can be found using magnitude and phase estimates of the correlation function at lags 0 and 1 (T_{PRF})

Autocorrelation method properties

- The ACM mean frequency estimate is unbiased for symmetric spectra, and approximatively so for narrow-band spectra

$$\text{Alternative formulation: } R(\tau) = e^{j\bar{\omega}_d \tau} \int_{-\pi}^{\pi} G(\omega) e^{j(\omega - \bar{\omega}_d)\tau} d\omega$$

The argument of R is equal to the (power-weighted) mean Doppler frequency ω_d if the imaginary part of the right side integral is zero:

$$\varepsilon = \int_{-\pi}^{\pi} G(\omega) \sin[(\omega - \bar{\omega}_d)\tau] d\omega = 0$$

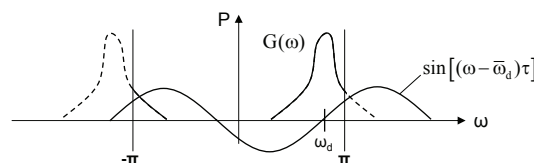
This odd sine-weighting implies that only symmetric spectra will fulfill this criterion. However, approximatively so also for practical Doppler spectra

Since white noise has a symmetric spectrum, this implies that the autocorrelation estimator is unbiased in presence of white noise

Reference: R. J. Doviak RJ and D. S. Zmic, *Doppler RADAR and weather observations*. San Diego, Academic press, Inc., 1993.

Autocorrelation method properties

- The mean frequency estimate is unbiased in the full Nyquist interval (for symmetric spectra)
 - Odd sine-weighting is centered at mean Doppler frequency



Initial mean frequency estimators proposed for use in multi-range Doppler and CFI suffered from drop-outs when the velocity of blood was approaching the Nyquist limit.

Reference: R. J. Doviak RJ and D. S. Zmic, *Doppler RADAR and weather observations*. San Diego, Academic press, Inc., 1993.

Autocorrelation estimator properties

- The variance of the autocorrelation estimator
Estimate correlation function by averaging over packet

- The variance of the power estimate *decreases* with increasing bandwidth

$$\text{var}(\hat{P}) = \frac{1}{N} \sum_{k=-N}^N \left(1 - \frac{|k|}{N}\right) |R(k)|^2 \approx \frac{P^2}{BN}$$

P = signal power, B = signal bandwidth,
N = ensemble (packet) size

- The variance of the mean frequency estimate *increases* with increasing bandwidth

$$\text{var}(\bar{\omega}_d) \sim \frac{B}{N}$$

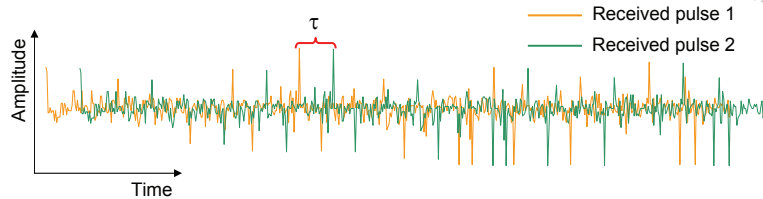
The variance of the autocorrelation estimates significantly improves by averaging the correlation estimates

Autocorrelation estimator properties

- Robust in low signal-to-noise environments
 - Superior to FFT-based method below ~15 dB, similar above ~15dB
- Computationally inexpensive
 - Ideally, in a noise free environment only two complex samples are needed to estimate the mean frequency
 - In practice more samples are needed to 1) attenuate clutter, and 2) reduce the variance of the correlation estimates

Cross-correlation method

- The velocity is proportional to the RF time shift between successive pulses



$$\tau = \frac{2\Delta z}{c} = \frac{2v \cos(\theta) T_{PRI}}{c} \quad \hat{R}_{12}(m, m_0) = \frac{1}{N_s} \sum_{k=0}^{N_s-1} r_1(m_0 + k) r_2(m_0 + k + m)$$

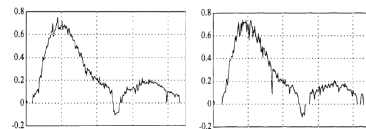
$$\hat{\tau}_{\max} = \arg \max \hat{R}_{12}(m) / F_s \quad \hat{v}_z = \frac{c}{2} \frac{\hat{\tau}_{\max}}{T_{PRI}}$$

Reference: O. Bonnefous and P. Pesque, *Time domain formulation of pulse-Doppler ultrasound and blood velocity estimation by cross correlation*, Ultrasonic imaging vol. 8, pp. 73-85, 1986.

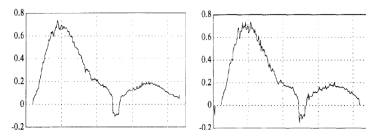
Properties of the cross-correlation method

- No aliasing under ideal circumstances
 - Signal decorrelation and lateral movement will limit this however
- Best performance for wide-band pulses
 - Higher resolution / lower penetration
- Axial sampling determines jitter error
 - Interpolation needed to find the true correlation maximum
- Time shift does not directly transfer to *mean-velocity*
- Computationally expensive compared to autocorrelation method

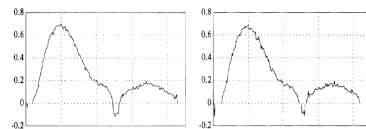
Auto correlation vs. Cross correlation method



Autocorr.
No averaging



Auto corr.
1.6us averaging



Cross corr.
1.6us averaging

Narrow band

Wide band

Example:

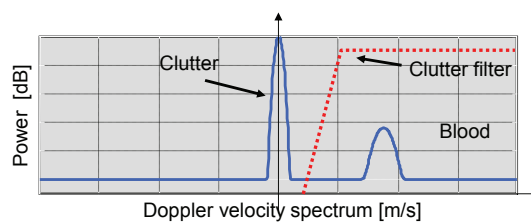
In vivo Comparison of autocorrelation and cross-correlation for data from the human subclavian artery

The two methods are approximately equal for narrow-band pulses and with radial averaging

Clutter filtering

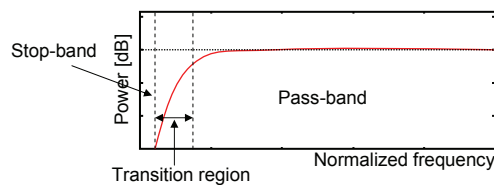
Clutter filtering

- Clutter is signal from surrounding tissue due to beam side lobes and reverberations
 - Can have 60-80 dB higher signal power than blood
- Blood typically has a higher velocity than tissue, i.e. higher Doppler shifts
 - The two components can thus be separated by high-pass filtering the Doppler signal



General clutter filter design

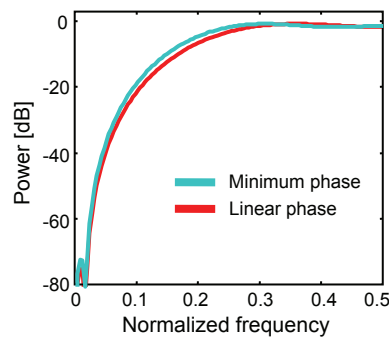
- Clutter filters should have a *high stop-band attenuation* (60-80 dB) to sufficiently attenuate clutter
- Clutter filters should have a *short transition region* to avoid removing signal from blood



- Several filter types have been used:
 - Finite impulse response (FIR) filters, Infinite impulse response (IIR) filters, and polynomial regression filters

Finite impulse response (FIR) filters

Example: Filter order $M=5$, packet size $N=10$



- The filter output y is the weighted sum of M past inputs x :

$$y(n) = \sum_{k=0}^{M-1} h(k)x(n-k)$$

- Discard the first M output samples, where M is equal to the filter order
- Improved amplitude response when nonlinear phase is allowed

Linear or non-linear phase response

- Conventional color flow imaging algorithms are based on the correlation function of the filtered signal

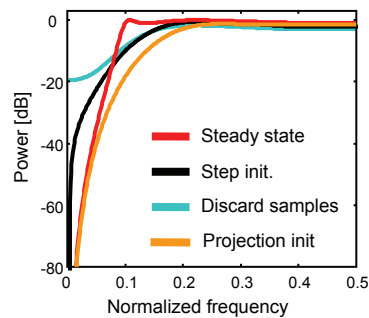
$$R_y(m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} S_x(\omega) \underbrace{|H(\omega)|^2}_{\text{Filter frequency response}} e^{jm\omega} d\omega \quad \text{(A linear transformation of the correlation function)}$$

Doppler spectrum Filter frequency response

- This means that it is safe to disregard the phase response when designing FIR clutter filters
- Without phase response constraints, an improved filter frequency response can be achieved for a given filter order
 - Minimum-phase response filters have the smallest time delay and most asymmetric impulse response

Infinite impulse response (IIR) filters

Example: Chebyshev order $M=4$, packet size $N=10$



- The filter output y is the weighted sum of past and current inputs x and past outputs y :

$$y(n) = -\sum_{k=1}^M a(k)y(n-k) + \sum_{k=0}^M b(k)x(n-k)$$

- Variations: Butterworth, Chebyshev, Elliptic
- Filter initialization* is critical to dampen the transient response (recursive filter)
- Chebyshev filter with projection initialization preferred for color flow imaging applications

1. E. S. Chornoboy, Initialization for improved IIR filter response, IEEE Trans. Signal processing, vol. 40(3), March 1992
2. S. Bjørn et al, Clutter Filter Design for Ultrasound Color Flow Imaging, IEEE Trans. Ultrason., Ferroelec., and Freq. Contr., vol. 49(2), pp. 204-216, Feb. 2002

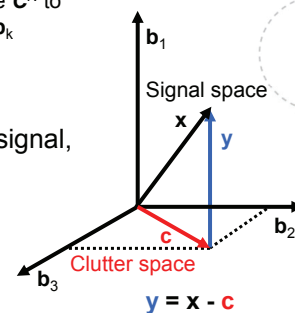
Regression filters

- The clutter c is estimated as a least squares fitting of the signal x to a set of basis functions b_k
 - A projection operation from the complex vector space C^N to the clutter subspace B^K spanned by basis functions b_k

$$c = \text{proj}_B(x) = \sum_{k=1}^K \langle x, b_k \rangle b_k$$

- The clutter component is subtracted from the signal, which can be formulated as a matrix operation

$$y = \left(I - \underbrace{\sum_{k=0}^{M-1} b_k b_k^{*T}}_{\text{Projection matrix}} \right) x = Ax$$



1. A. P. G. Hoeks et al. An efficient algorithm to remove low frequency signals in digital Doppler systems. Ultrasound Imaging, vol. 13, pp. 135-144, 1991
2. H. Torp, Clutter rejection filtering in Color Flow Imaging: a theoretical approach, IEEE Trans. Ultrason., Ferroelec., and Freq. Contr., vol. 44(2), pp. 417-424, Feb. 1997

Choice of basis functions

- The choice of clutter basis functions has a large impact on performance

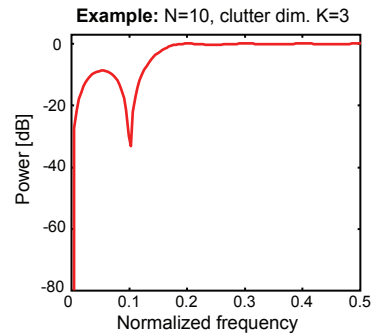
Fourier basis:

$$b_k = \frac{1}{\sqrt{N}} e^{jkn/N}, \quad n = 0, \dots, N-1$$

Orthonormal and equally distributed in frequency

Analogy:

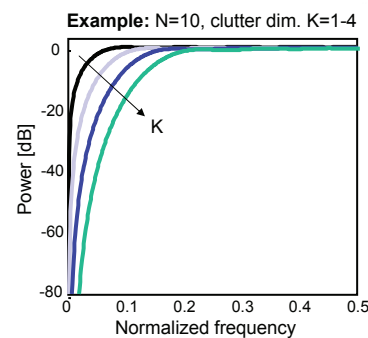
1. Discrete Fourier transform
2. Set low frequency coefficients to zero
3. Inverse Fourier transform



Legendre polynomial basis

- A set of orthonormal polynomials: $\int_{-1}^1 P_m(x) P_n(x) dx = \delta_{mn}$

$$\begin{aligned}
 &b_0 = \text{---} \\
 &+ \\
 &b_1 = \text{---} \\
 &+ \\
 &b_2 = \text{---} \\
 &+ \\
 &b_3 = \text{---} \\
 &+ \\
 &\vdots \\
 &y = \left(I - \sum_{k=0}^{M-1} b_k b_k^{*T} \right) x = Ax
 \end{aligned}$$

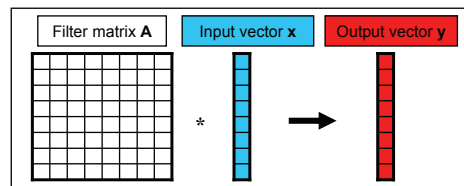


The Legendre polynomials can be generated through a Gram-Schmidt orthonormalization procedure on the polynomials x^k , $k=0, \dots, K$

Linear clutter filter formulation

Why should clutter filters be linear?

- No intermodulation between clutter and blood signal
- Preservation of signal power from blood
- Optimum detection (Neuman-Pearson test) includes a linear filter



Any linear filter can be performed by a matrix multiplication of the N -dimensional signal vector x

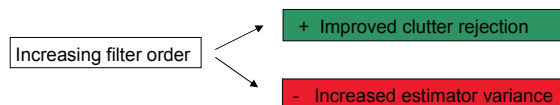
$$y = Ax$$

This form includes all IIR filters with linear initialization, FIR filters, and regression filters

FIR filter matrix structure

$$A = \begin{bmatrix} b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & 0 & . & . & 0 \\ 0 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & . & . & . \\ . & . & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & . & . \\ . & . & . & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 & 0 \\ 0 & . & . & 0 & b_1 & b_2 & b_3 & b_4 & b_5 & b_6 \end{bmatrix}$$

FIR filter order	M=5
Packet size	N=10
Output samples	N-M= 5



Frequency response of linear filters

- The frequency response of linear filters is defined as the filter output for a single frequency complex sinusoid input

$$H_0(\omega) = \frac{1}{N} \|A\mathbf{e}_\omega\|^2, \quad -\pi < \omega < \pi \quad \mathbf{e}_\omega = [1 e^{j\omega} \dots e^{j(N-1)\omega}]^T$$

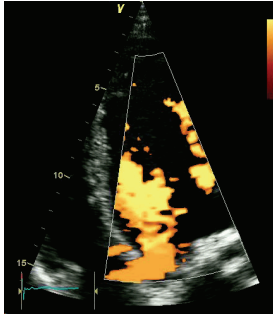
Note: The output of the filter is not in general a single frequency signal. This is only the case for FIR filters.

Note: Frequency response is only well-defined for complex signals

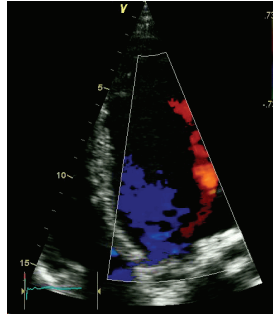
Reference: H. Torp, *Clutter rejection filtering in Color Flow Imaging: a theoretical approach*, IEEE Trans. Ultrason., Ferroelec., and Freq. Contr., vol. 44(2), pp. 417-424, Feb. 1997

CFI display algorithms

Color mapping types



Power Doppler (Angio)
Brightness ~ signal power



Color flow
Brightness ~ signal power
Hue ~ Velocity



Color flow "Variance map"
Hue & Brightness ~ Velocity
Green ~ signal bandwidth

Image example : Aortic regurgitation

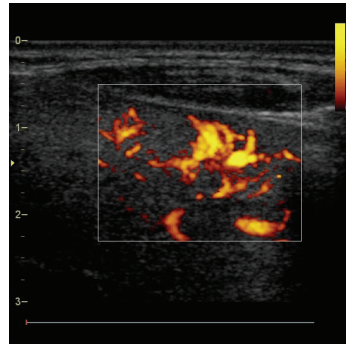
Tissue / flow arbitration

- B-mode and CFI image acquired separately due to the resolution / penetration requirements / compromises in CFI and B-mode
- The two images are typically combined through a hard decision of whether to display a B-mode or color pixel → *Arbitration*
- Post-processing is also needed in order to reduce the amount of flashing artifacts due to insufficient clutter attenuation

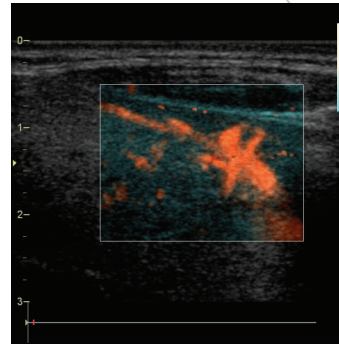
This decision is typically based on:

- Mean frequency and power before and after filtering
- High power / low frequency (below filter cut-off) => tissue

Arbitration examples



Power-Doppler: Hard arbitration



Power-Doppler: Soft arbitration

Example: Thyroid nodule vascularization

Topics not covered here

- Advanced axial velocity estimators
 - Techniques utilizing 2-D signal information
- Synthetic transmit aperture flow imaging
 - Lots of interesting aspects investigated
- Coded excitation in color flow imaging
 - When is it feasible?
- B-Flow imaging
 - Simultaneous acquisition of B-mode and flow images
 - Coded excitation to retain both resolution and penetration
- And more...



Doppler Short Course '09

Estimation and imaging blood flow velocity
Part III: Advanced topics, trade-offs and limitations

2009 Ultrasonics Symposium, Rome, Italy

Hans Torp, Lasse Løvstakken

Department of Circulation and Medical Imaging
 Norwegian University of Science and Technology

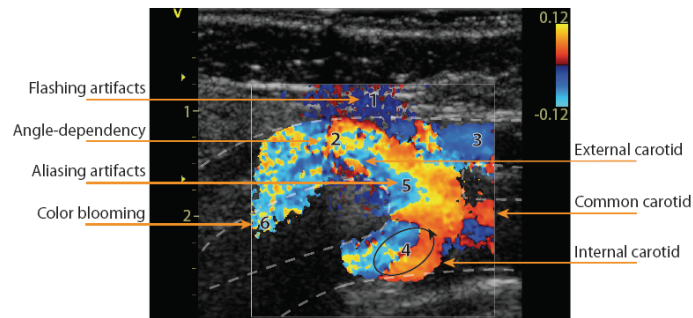


Limitations of current Doppler / CFI techniques



Limitations of CFI

- Conventional methods suffer from several fundamental limitations that restrict its use in clinical practise



Data acquisition limitations

- Sufficient signal-to-noise ration vs. spatial resolution
 - The spatial resolution typically needs to be lowered compared to B-mode to achieve a sufficient SNR
 - This leads to *color blooming artifacts* in CFI where the color image smears into the B-mode image
- Frame rate vs. estimator quality vs. lateral sampling
 - The packet size and number of image lines is reduced in order to achieve a sufficient frame rate for following the flow dynamics
 - In real-time 3-D CFI, frame rate is a particular challenge

Clutter filtering limitations

- Clutter is often not properly attenuated during the whole cardiac cycle
 - This leads to *flashing artifacts* where tissue regions are falsely colored as blood
- Clutter filtering performance vs. packet size
 - Due to frame rate requirements, the number of samples available for clutter filtering is limited → long filter transition regions
- Clutter effect on velocity estimates
 - Signal varying bias towards zero if not properly attenuated
 - The clutter filter may also remove substantial blood signal if the angle is near transverse

Velocity estimator limitations

- Only the axial velocity component is measured
 - Measured velocities depend on the angle between the ultrasound beam and vessel of interest → *angle-dependency artifacts*
 - For transverse flow, the mean Doppler shift is centered around zero, and a substantial part is removed by the clutter filter
- Aliasing artifacts
 - A limited velocity range can be measured before *aliasing* occurs, determined by the PRF
 - Using a higher PRF increases the maximum limit, but sensitivity to low velocities is then reduced (in practice due to clutter filter)
- Bias / variance
 - The bias and variance can in practice be quite high, and varies in space and time → the use of CFI has mainly been of qualitative nature

Patient safety in Doppler imaging

Patient safety in Doppler imaging

- Potential hazardous heating and mechanical effects restrict the allowed acoustic output of ultrasonic imaging equipment
- Acoustic output is restricted by one of the following:
 - Mechanical index (MI), a measure of mechanical effects
 - Spatial peak temporal average intensity (I_{spta}), total output power, or thermal index (TI)
 - Transducer surface temperature
- Heating effects are averaged for combinations of modes (duplex / triplex modes). Mechanical effects are determined by the modality with the highest value.

The outcome: The sensitivity / penetration in Doppler modes may be severely punished from these restrictions

Mechanical effects

- Mechanical effects are related to cavitation, i.e. the formation and collapse of gas bubbles

Mechanical index: $MI = \frac{p_{r,3}(z_{sp})}{\sqrt{f_{awf}}}$

$\xrightarrow{\text{Derated peak rarefactional pressure}}$
 $\xrightarrow{\text{Acoustic working frequency}}$

Focal gain: $G_{foc} = \frac{D_a D_e}{R_{foc} \lambda} = \frac{D_e}{F_{\#} \lambda} = \frac{D_e \cdot f_0}{F_{\#} \cdot c}$ (Coinciding ax. an el. Foci)

- MI is measured at the axial point of maximum *derated* temporal averaged intensity (I_{ta})
- **MI is effectively proportional to applied voltage, sqrt(f_0), and inverse proportional to the F-number**

Tissue heating

- Heating occur in the tissue due to absorption, at a rate much lower (~seconds) than the ultrasound frame rate
- Spatial-peak temporal average intensity (I_{spta}):**

$$I_{spta} = \max_z \left[PRR \cdot \int_0^{T_{PRR}} p_z^2(t) / \rho c \, dt \right]$$

- Proportional to the transmitted pulse energy
- Proportional to the squared focal gain, G_{foc}
- For scanned modes, the intensity contributions for overlapping beams are added

Transducer surface heating

- **FDA regulations:** The transducer surface may not exceed 43 deg. held towards the skin, or 50 deg. towards air
- For current transducer technology, surface heating is almost always higher than at the geometric focus¹
→ *It is typically the most limiting factor with regards to voltage / PRF / apertures*
- The temperature rise for duplex and triplex modes (B-mode+CFI+PW-Doppler) is an average for a common time constant (frame rate / PRF)

Reference: ¹M. Curley, *Soft Tissue Temperature Rise Caused by Scanned, Diagnostic Ultrasound*, IEEE Trans., Ultrason., Ferroelect., and Freq. Contr., vol. 40(1), Jan 1993

Transducer surface heating

- Empirical model of surface temperature:

$$\Delta T(x) = \sum_{n=1}^{N_e} W_n \times \Delta T_n(x)$$

Temperature increase for element n at position x along the surface

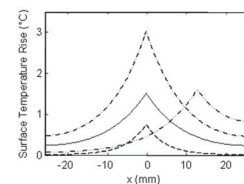
The number of times element n is fired

$$\Delta T_n(x) \sim \frac{\text{PRF} \cdot V^2 \cdot \bar{\beta}(f_0, B_0)}{2\alpha} \cdot g(x | x_n, D_{ap}, \alpha)$$

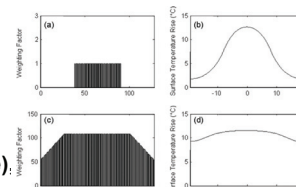
Legend: PRF = pulse repetition frequency, V = voltage over element, α = measured thermal conductivity / efficiency parameter, β = measured pulse parameter, D_{ap} = aperture width, x_n = element x position

Temperature rise is proportional to PRF, V^2 , $g(\text{aperture})$, pulse length, ...

Reference: W.-S. Ohm et al, *Prediction of Surface Temperature Rise of Ultrasonic Diagnostic Array Transducers*, IEEE Trans., Ultrason., Ferroelect., and Freq. Contr., vol. 55(1), Jan 2008



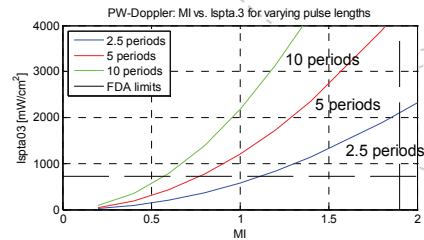
Temperature rise $\Delta T_n(x)$ for different transducer elements



Different weighting factors, W_n .
Top: PW-Doppler, bottom: scanned

PW-Doppler acoustics

- For a nonscanning situation, the maximum $Ispta$ is close to the focal region
- The longer pulses typically used in PW-Doppler significantly increases heating effects
- For duplex operation (B-mode + PW-Doppler), surface temperature increase is mostly dominating
- For triplex operation (B-mode+CFI+PW-Doppler), $Ispta$ or surface temperature will dominate

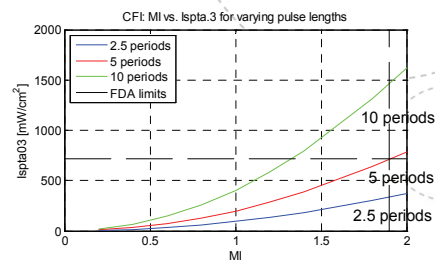


Setup: Linear array, $f_0 = 5\text{MHz}$, $FN_{tx} = 2.5$, $txFocus = 4\text{ cm}$, $PRF = 8\text{kHz}$

Outcome: PW-Doppler is typically $Ispta$ limited, exceptions include very low PRFs, low frequencies, and narrow focusing

Color Flow Imaging acoustics

- For a scanning situation, the maximum $Ispta$ is close to the surface
- Similar characteristics for phased-array (cardiac) probes as for linear array (vascular) probes
- Duplex operation adds heating effects from B-mode
- Generally surface temperature limited
- MI limited only for short pulses, low frequencies and PRFs, narrow focusing



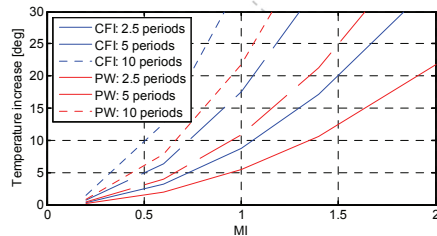
Setup: Linear array, $f_0 = 5\text{MHz}$, $FN_{tx} = 2.5$, $txFocus = 4\text{ cm}$, $PRF = 4\text{kHz}$, 2 cm ROI width , 50 beams . **NB: B-mode values not added**

CFI is typically transducer surface temperature limited.

Exceptions include very low PRFs, low frequencies, and narrow focusing

Surface temperature comparison

- An example of surface temperature prediction for PW vs. CFI for the same aperture / tx-focus
- CFI generates a higher surface temperature than PW-Doppler due to:
 - Aperture overlap during scanning, i.e. individual elements are excited more often,
 - The imaging PRF is often higher in CFI than for PW-Doppler



Common setup: Linear array, $f_0 = 5\text{MHz}$, $\text{FNtx} = 2.5$, $\text{txFocus} = 4\text{ cm}$

CFI: $\text{PRF}_{\text{img}} = 16\text{kHz}$, $\text{PRF} = 4\text{kHz}$, $N_p = 12$, 2 cm ROI width, 50 beams.

PW: $\text{PRF}_{\text{PW}} = 8\text{kHz}$, $N_p = 64$

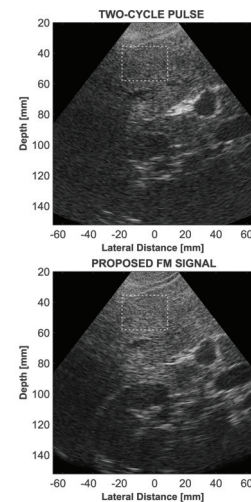
NB: B-mode values not added

Patient safety summary

- Transducer surface heating is currently the main limitation in CFI and B-mode+PW-Doppler, while I_{spta} is the main limitation in CW- and PW-Doppler (without B-mode)
 - MI limitations might occur for low PRFs, short pulses, and narrow focusing
- For an equal PRF, the temporally average transmitted energy is distributed over a larger spatial region for scanned modes than for PW-Doppler:
 - Lowering overall tissue heating effects for scanned modes
 - Bringing the maximum temperature close to the transducer for scanned modes
 - Especially the case for 3-D imaging
- These rules of thumb apply to both linear array (vascular) as well as phased array (cardiac) imaging

Coded excitation in Doppler

- Using *non-linear phase-modulation* can increase the pulse length without reducing the bandwidth, and may therefore be used to increase the amount of transmitted energy
- *Deconvolution filtering* on receive restores the phase to retain axial resolution
- The theoretical improvement in SNR is equal to the *time-bandwidth product* of the transmitted waveform, *TB*.
 - I.e., for a given bandwidth, doubling the pulse length by coding may theoretically double the SNR



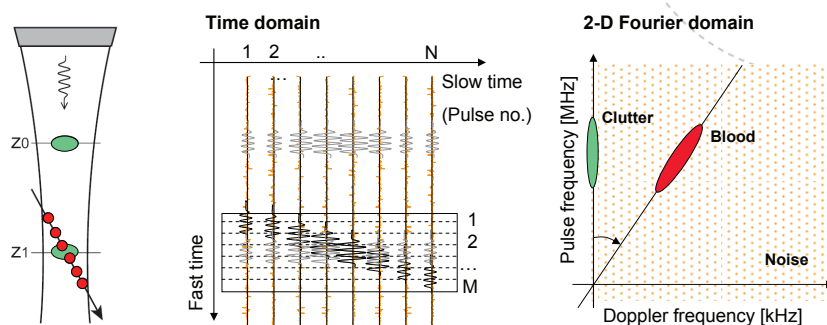
Reference: T. Misaridis and J. A. Jensen, *Use of Modulated Excitation Signals in Medical Ultrasound. Part I: Basic Concepts and Expected Benefits*, IEEE Trans., Ultrason., Ferroelect., and Freq. Contr., vol. 52(2), Feb 2005

When is coded excitation applicable in Doppler modes?

- Codes are in general applicable only when MI-limited
- As previously indicated, heating effects are typically prominent and will severely limit the advantage of codes in Doppler modes
- Disregarding surface heating, a potential might however exist for codes in CFI
 - More efficient transducer materials
 - Active cooling
 - Unfocused imaging ($MI \sim G_{foc} \cdot I_{spta} \sim G_{foc}^2, \rightarrow G_{foc} = 1$)

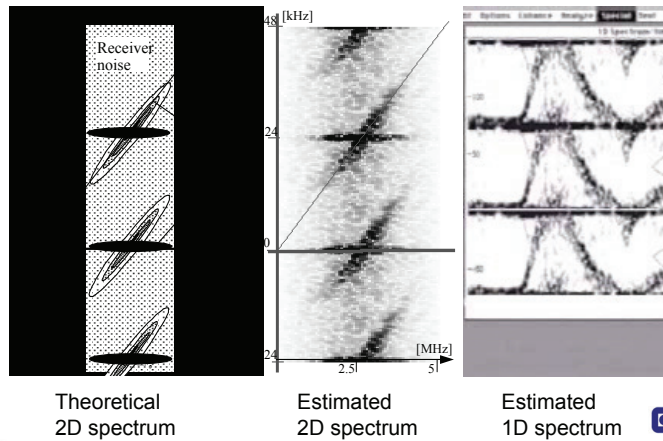
Advanced axial velocity estimators

2-D Doppler signal representation

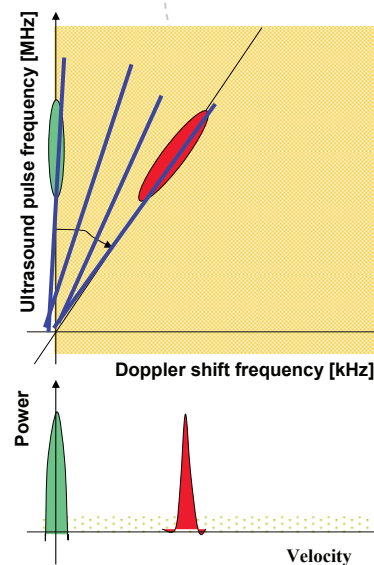
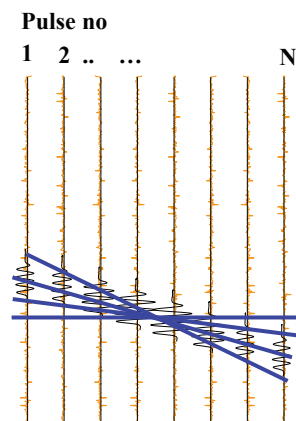


This 2-D representation contains the total signal information available for processing

2-D spectrum example

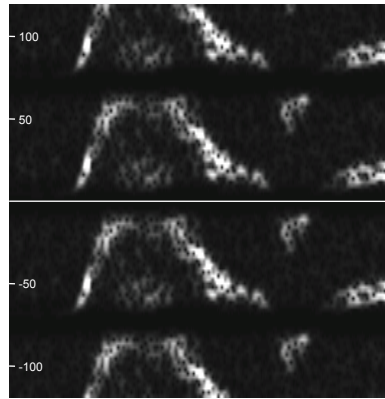


Velocity matched spectrum algorithm for PW-Doppler

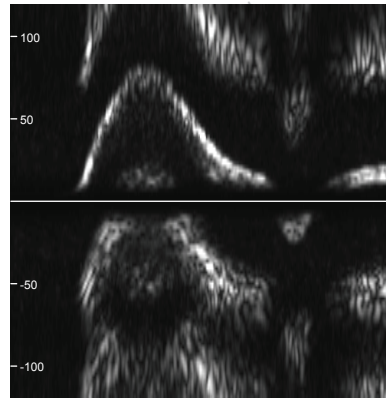


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Blood velocity spectrum Subclavian Artery



Conventional spectrum



Velocity matched spectrum

www.ntnu.no

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Advanced velocity estimation in Color Flow Imaging

- The autocorrelation approach is a **narrow-band technique** that only utilizes a portion of the signal information available
- Several **wide-band velocity estimators** have been proposed, some examples:
 - Cross-correlation technique
 - 2-D spectral processing techniques
 - Wide-band maximum likelihood estimator
- These methods may outperform the ACM with regards to accuracy and aliasing limit

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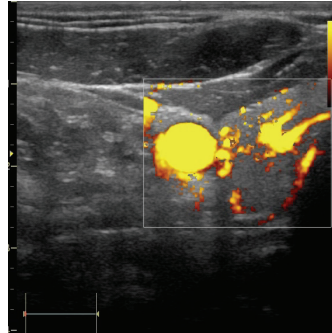
Disadvantages of wide-band processing

- Wide-band, i.e. short pulses are needed, which may lead to SNR problems
- Decorrelation limits the practical velocity range that may be tracked beyond the Nyquist range
- Substantially increased computational load

Adaptive clutter filtering

Adaptive clutter filtering in CFI

- An adaptive filter has a data dependent response
- Adapting the clutter filter to clutter characteristics may:
 - Reduce flashing artifacts by more effectively attenuating clutter
 - Remove less blood signal throughout the cardiac cycle
- Especially relevant in:
 - Low-flow environments
 - Excessive tissue movement environments

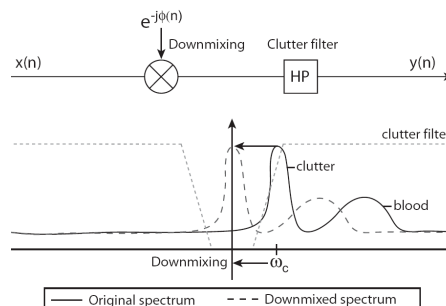


Example: Flashing artifacts when imaging an healthy carotid artery and thyroid (low PRF)

The downmixing approach

Algorithm:

- Estimate the main tissue Doppler frequency, ω_t
- Downmix the signal with ω_t prior to conventional clutter filtering



Reference: L. Thomas and A. Hall, An improved wall filter for flow imaging of low velocity flow, Proceedings of the IEEE Ultrasonics Symposium, 1994

The downmixing approach

- The clutter Doppler frequency is obtained using the autocorrelation approach on spatially averaged data
- Downmixing can be done based on the estimated *mean frequency* of clutter, or the *instantaneous frequency* of clutter

Mean frequency:

$$\phi_{mf}(n) = \hat{\omega}_c n = \angle \left[\sum_{k=1}^{N_p-2} \hat{R}(k,1) \right] n$$

Instantaneous frequency:

$$\phi_{if}(n) = \begin{cases} 0 & n = 0 \\ \left[\sum_{k=1}^n \angle \hat{R}(k,1) \right] & n = 1, \dots, N-2 \end{cases}$$



Reference: S. Bjærum et al, Clutter Filter Design for Ultrasound Color Flow Imaging, *IEEE Trans. Ultrason., Ferroelec., and Freq. Contr.*, vol. 49(2), pp. 204-216, Feb. 2002

The downmixing approach

- Downmixing filters are in general quite robust, however:
 - Stationary tissue signal can be shifted *into* the pass band of the clutter filter
 - Instantaneous frequency downmixing may lead to *artifacts* in the shifted Doppler spectrum
- Downmixing with the *instantaneous* clutter Doppler frequency provides superior clutter suppression
- A *combined approach* is possible where:
 - Detection and arbitration of blood is done with instantaneous frequency downmixing and filtering
 - Velocity estimation is performed after mean frequency mixing and filtering

The eigen-based concept

Background

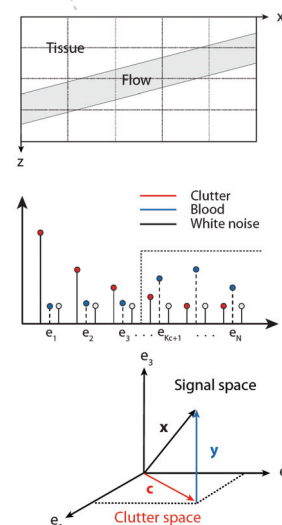
- The clutter contribution to signal variation is different then from blood
Typically: The clutter component is more dominant (higher energy) and infers more low-frequency variation than blood

Filter principle

- Principal component analysis (PCA) / the Karhunen Loeve expansion (KLE) separates the main sources of variation
- By selecting the modes of variation based on *a priori* knowledge of clutter, a signal basis can be found and used to attenuate clutter

Eigen-based algorithm

1. Estimate correlation matrix
Average in a spatial region
2. Signal decomposition
Estimate eigenvectors and eigenvalues, i.e. the orthogonal signal basis functions in PCA
3. Identify clutter eigen-components
Based on a priori assumptions
For instance the most dominant in energy
4. Form filter matrix and filter
Regression / projection filtering approach



The eigen-based formulas

- Signal model consists of up to *three independent components*, clutter c , noise n , and optionally blood b

$$x = c + n + b \quad R_x = R_c + R_n + R_b$$

- The signal correlation matrix is estimated by **spatial averaging**

$$\hat{R}_x = \frac{1}{M} \sum_{m=1}^M x_m x_m^{*T}$$

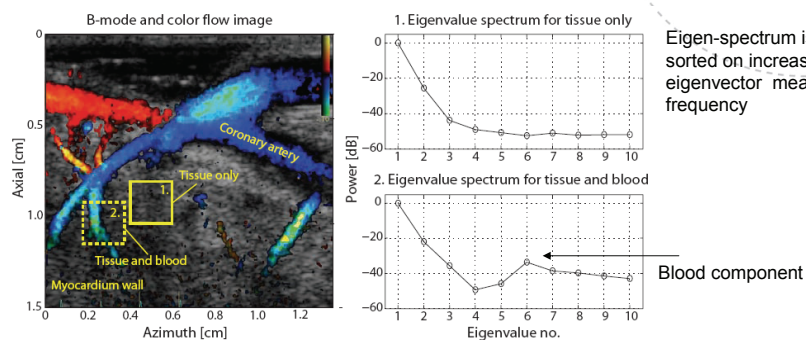
- The clutter is represented by the orthonormal basis given by **selected eigenvectors** of the signal correlation matrix

$$x = \sum_{k=1}^{N_D} \gamma_k e_k, \quad E\{\gamma_k \gamma_l^*\} = \begin{cases} \lambda_k, & k=l \\ 0, & k \neq l \end{cases}$$

- Filtering is performed as for regression filters through **projection**

$$P = I - \sum_{k=1}^{K_c} e_k e_k^{*T}$$

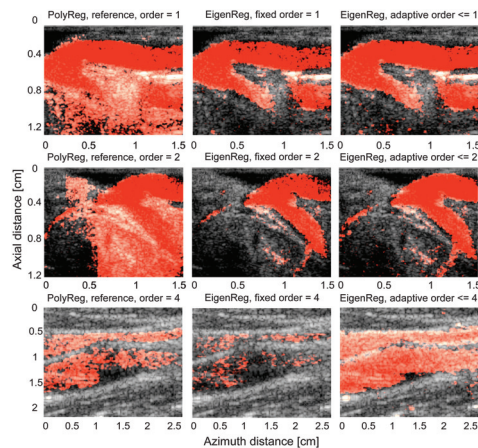
An eigen-spectrum example



Eigen-spectrum for 1) tissue+noise, and 2) tissue+blood+noise

Image example: Epicardial imaging of porcine myocardium at 14 MHz

Eigen-based filter examples



Clutter filtering examples:

- 1-2: Coronary artery bypass
- 3: Carotid artery

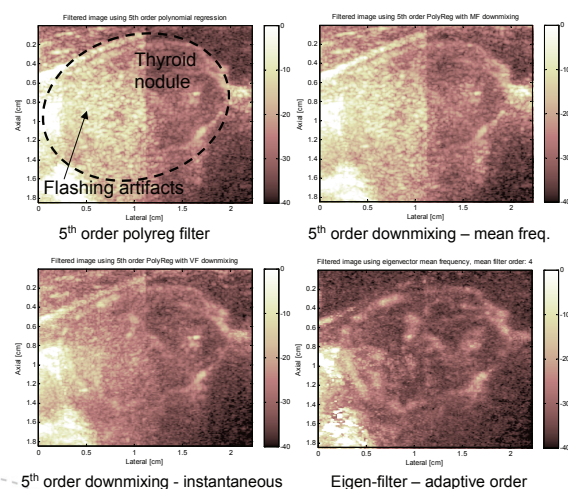
All example frames are from the systole with maximum tissue movement. 40 dB dynamic range for tissue, 6 dB dynamic range for flow

Observations:

1. Eigen-based method provides an improved suppression of clutter
2. If too many eigenvectors are included in clutter basis, then blood signal is lost

Reference: L. Løvstakken et al, Real-time adaptive clutter rejection filtering in color flow imaging using power method iterations, *IEEE Trans. Ultrason., Ferroelec., and Freq. Contr.*, vol. 53(9), pp. 1597-1608, Sept. 2006

Downmixing vs. eigen-approach



Example:

Thyroid gland tumor nodule, raw power-Doppler image, PRF = 250 Hz (extreme)

General observations:

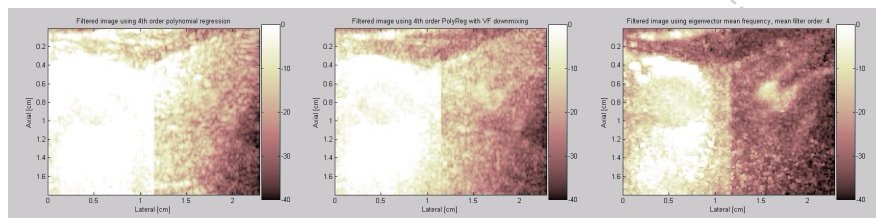
The eigen-approach achieves superior clutter suppression for lower order filters

The eigen-approach is dependent on an adaptive selection of filter order

Instantaneous downmixing performs quite well in general

The downmixing filters in general seem more robust

Comparison movies

4th order polyreg filter4th order downmixing - instantaneous

Eigen-filter – adaptive order

Example:

Thyroid gland tumor nodule, raw power-Doppler image, PRF = 250 Hz (very low)

Summary – adaptive clutter filtering

- Eigen-based filtering can provide superior clutter suppression for low filter orders in regions with excessive tissue motion, however:
 - The filter order must be chosen adaptively for different regions of the image for robust performance
 - Spatial averaging of the correlation matrix estimate smoothes the resulting flow images
- Downmixing filtering can provide an improved suppression of clutter in general, however:
 - Stationary signal may be shifted into the pass band
 - Artifacts may appear in the Doppler spectrum when using a varying (instantaneous) downmixing frequency



Doppler Short Course '09

Estimation and imaging of blood flow velocity

Part IV: Future directions

2009 Ultrasonics Symposium, Rome, Italy

Hans Torp and Lasse Løvstakken

Department of Circulation and Medical Imaging

Norwegian University of Science and Technology



Lecture outline

- 2-D vector velocity imaging of blood
 - Vector-Doppler, speckle tracking, echo-PIV
- High frame rate Color Flow Imaging
 - Parallel beamforming, plane-wave imaging, synthetic aperture flow imaging, multi-transmit CFI
- Real-time 3-D ultrasound applications
 - 3-D color flow imaging, high-PRF 3-D imaging of mitral valve leakage



2-D velocity vector imaging

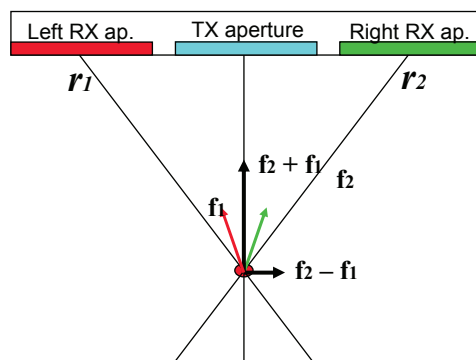
2-D blood vector velocity mapping

- Vector-Doppler techniques
 - Combine Doppler shift from spatially separated transmit / receive apertures
- Speckle imaging / tracking techniques
 - Visualization of speckle pattern movement
 - Tracking of speckle pattern movement
- Transit-time broadening techniques
 - The bandwidth of the Doppler signal is related to the lateral velocity component
- Synthetic aperture techniques
 - Beamforming and cross-correlation along arbitrary lines. Choosing the direction of maximum correlation

Vector-Doppler techniques

- Compound Doppler techniques
 - Combine Doppler shift from several physically / electronically separated transmit / receive apertures
- Lateral oscillation techniques
 - Common transmit aperture, spatially separated receive apertures created electronically using “non-uniform” apodization functions
 - Transverse oscillation method (TO) (Jensen 1998)
 - Spatial quadrature method (SQ) (Anderson 1998)

Compound Doppler method with common transmit beam



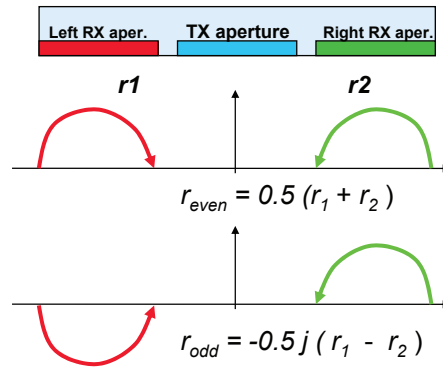
f_1 : Doppler shift left aperture
 f_2 : Doppler shift right aperture

Velocity components:

$$V_x = (\lambda/2) (f_2 - f_1)$$

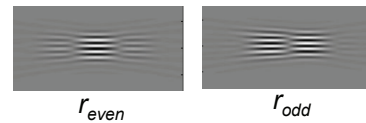
$$V_z = (\lambda/2) (f_2 + f_1)$$

Spatial quadrature technique



An oscillation in the lateral field is created using a non-uniform receive apodization function (left/right receive)

The left / right receive apertures are separated using quadrature relations

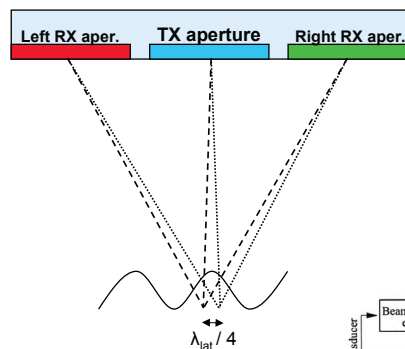


Reconstruction:

$$\begin{aligned} r_1 &= r_{\text{even}} + j r_{\text{odd}} \\ r_2 &= r_{\text{even}} - j r_{\text{odd}} \end{aligned}$$

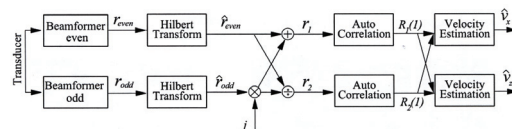
Reference: M. E. Anderson. *Multi-dimensional velocity estimation with ultrasound using spatial quadrature*. IEEE Trans., Ultrason., Ferroelect., and Freq. Contr., vol. 45(3), pp. 852-861, May 1998

Transverse oscillation technique



An oscillation in the lateral field is created using a non-uniform receive apodization function (left/right receive)

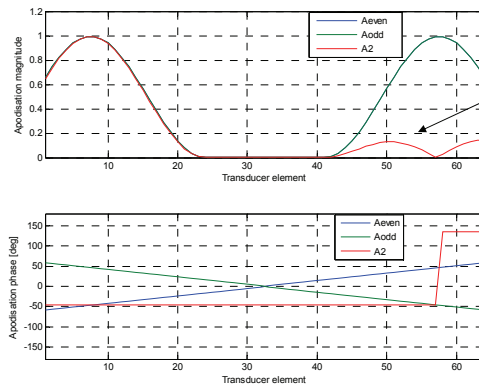
The left / right receive apertures are separated by *sampling beams one quarter lateral wavelength apart*



References:

J. A. Jensen and P. Munk. *A new method for estimation of velocity vectors*. IEEE Trans., Ultrason., Ferroelect., and Freq. Contr., vol. 45(3), pp. 837-851, May 1998
J. Udesen and J. A. Jensen. *Investigation of Transverse Oscillation Method*. IEEE Trans., Ultrason., Ferroelect., and Freq. Contr., vol. 53(5), pp. 959-971, May 2006

Compound Doppler vs TO comparison



Separation error

- SNR-ratio analysis:
TO method - 0.11 dB
- Mirror spectral energy: 29 dB
- No significant velocity bias
when using autocorrelation
method

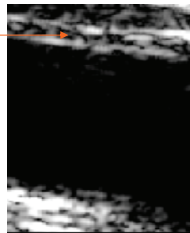
Summary of vector Doppler methods

- *Spatial quadrature* and *compound Doppler* are identical methods
- *Transverse oscillation* method is approximately equal to *compound Doppler* method for small apertures and narrowband pulses
- All three methods give approximately equal performance for typical aperture / pulse bandwidths

Blood speckle imaging

- The speckle pattern from blood flow signal is isolated by high-pass filtering the Doppler signal
- The movement of this speckle pattern is correlated to the movement of blood

Tissue echo
speckle pattern



Blood flow echo
speckle pattern

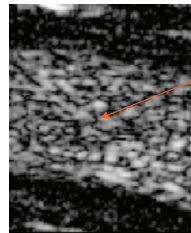


Figure: B-mode image of carotis artery and corresponding filtered image showing the blood flow speckle pattern utilized in BFI

Blood Flow Imaging

Speckle movement visualization

- Speckle pattern images of moving blood is captured at a **high frame rate**, and is played back in **slow motion**
- The movement of the blood flow speckle pattern can then be **visually tracked from image to image**

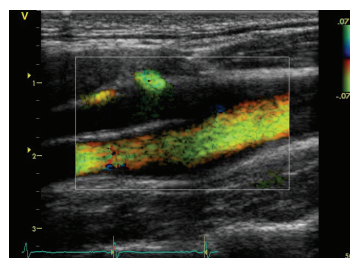
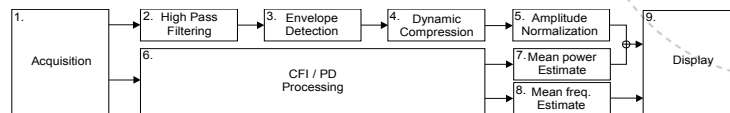


Figure: A visualization of the speckle pattern movement from blood in a carotid bifurcation

Speckle images are superimposed onto conventional Doppler images

BFI - signal processing



- High-pass filtering produce signal from blood
 - Time-invariant FIR filters preserve speckle pattern similarities
- B-mode processing produces speckle images
- Amplitude normalization
 - Reduce signal power discontinuities between frames
- Combine with CFI
 - Modulate the parametric color images with speckle pattern images
 - Interpolate color images in time to match the number of speckle images

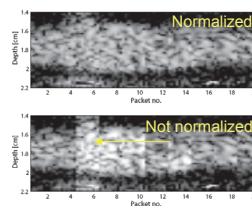
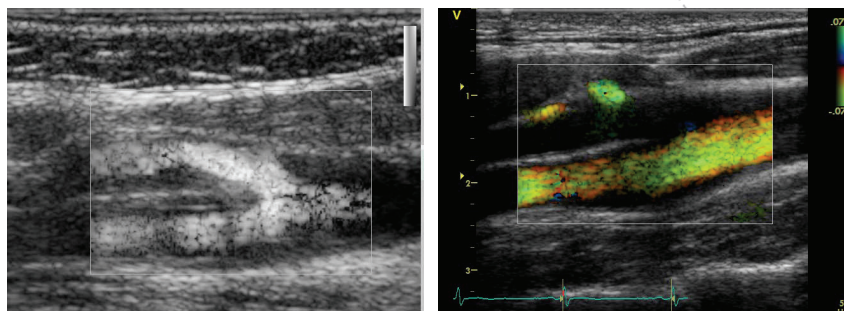


Figure: Normalization needed to reduce discontinuities between image frames

BFI example 1 – vascular imaging



Vascular imaging

Imaging of a stenosed carotid bifurcation

Left: Power Doppler + speckle, right: CFI + speckle

BFI example 2 – coronary bypass surgery

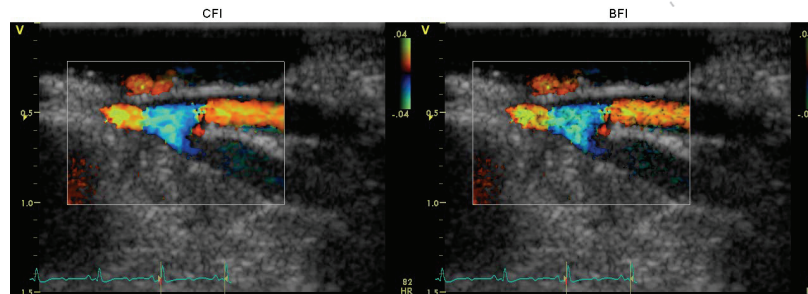


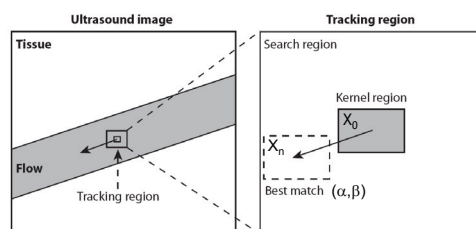
Figure 6.3: Imaging of a proximally snared anastomosis (standard case) using CFI and BFI respectively.
L. Lovstakken, K. S. Ibrahim, N. Vitale, S. T. Hendricken, I. Kirkeby-Garstad, H. Torp, and R. Haaverstad,
"Blood Flow Imaging – A new 2-D ultrasound modality for enhanced intraoperative visualization of blood flow patterns in coronary anastomoses"

Intraoperative imaging in cardiothoracic surgery

Imaging of a proximally occluded (snared) LIMA-LAD bypass anastomosis, using CFI (left) and BFI (right)

Speckle tracking of blood

- Tracking speckle pattern between subsequent frames, speckle displacement is given by best match



Sum of Absolute Differences (SAD):

Block matching algorithm

Real-time operation feasible (special purpose multimedia instructions)

Displacement given for minimal SAD:

$$\varepsilon_{\min} = (\alpha_m, \beta_m)$$

$$\varepsilon(\alpha, \beta, n) = \sum_{i=1}^I \sum_{j=1}^k |X_0(i, j) - X_n(i + \alpha, j + \beta)|$$

$$V_n = \frac{\sqrt{(\alpha_m \Delta x)^2 + (\beta_m \Delta z)^2}}{nT}$$

$$g_n = \arctan \frac{\alpha_m \Delta x}{\beta_m \Delta z}$$

G. E. Trahey et al. Angle Independent Ultrasonic Detection of Blood Flow. IEEE Trans., Biomed. Eng., vol. 34(12), pp. 965-967, Dec 1987

L. N. Bohs et al. Speckle tracking for multi-dimensional flow estimation. Ultrasonics, vol. 38, pp. 369-375, 2000

Speckle tracking challenges

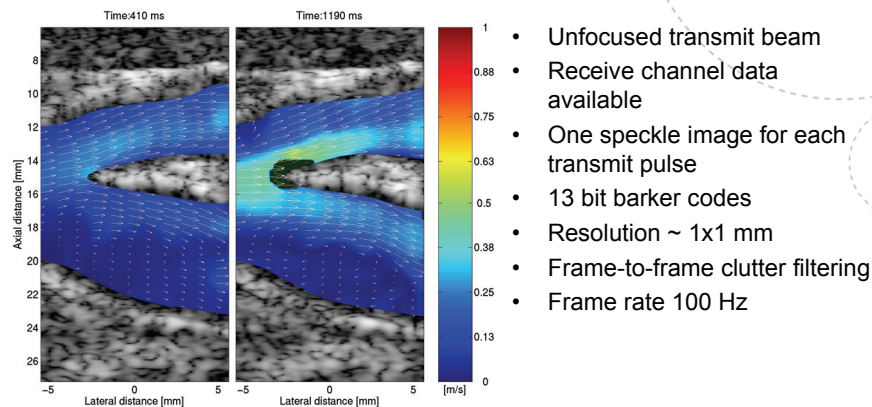
- **Speckle decorrelation**
The speckle pattern decorrelates rapidly due to:
 - a) Spatial and temporal velocity gradients
 - b) Non-laminar flow
 - c) Out-of-plane movements
- **Clutter filtering**
 - Doppler-based high-pass filtering removes blood signal for near-transversal flow
 - Reduces SNR, and influences the speckle appearance

Speckle tracking acquisition

A *very high frame rate* is needed due to the rapid decorrelation

- **Interleaved / parallel beam acquisition**
 - Speckle region frame rate equal to the user chosen PRF
 - Two-way resolution / penetration
 - Time lag across image as in conventional color flow imaging
- **Plane wave acquisition**
 - Single unfocused transmit beam, all receive channels available
 - Continuous acquisition, processing from frame to frame
 - High overall frame-rate (~ 100 Hz), no time lags across image
 - Reduced resolution and penetration $\rightarrow$ coded excitation

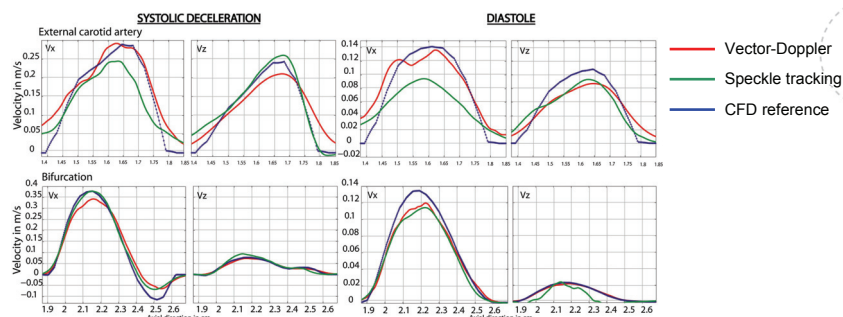
Plane wave emission



J. Udesen et al, "Fast Blood Vector Velocity Imaging: Simulations and Preliminary in vivo Results", Proceedings of IEEE Ultrasonics Symposium, New York 2007

Speckle tracking vs. vector-Doppler

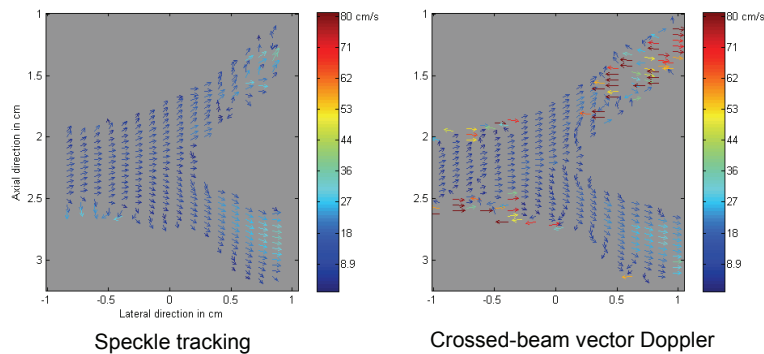
- By simulating ultrasound signals from realistic flow fields obtained using computational fluid dynamics, experimental methods can be evaluated and compared statistically in a controlled manner



A. Swillens et al., Ultrasonics Symposium 2008, Beijing, China

Speckle tracking vs. crossed-beam vector Doppler

- Simulated data from a carotid artery bifurcation model (CFD)
- Comparison movie for a clutter filtered example at 2kHz PRF



A. Swillens et al. Oral presentation given at blood flow session Wednesday, 23. Aug.

Clutter filtering in 2-D velocity estimation

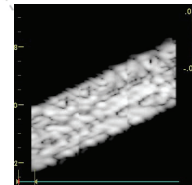
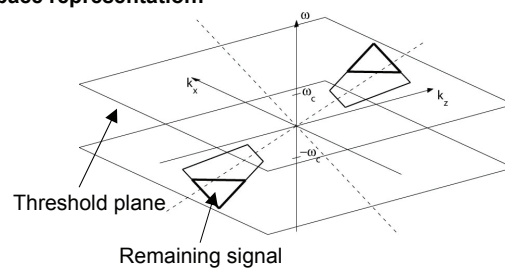
- Doppler-based clutter filtering will remove signal from blood as the beam-vessel angle approaches 90°
- Vector-Doppler
 - Signal drop-out for tx/rx aperture from one side
- Speckle tracking
 - Signal drop-out for near transversal flow
 - Remaining signal becomes band pass also laterally → lateral oscillations

One of the main challenges in 2-D velocity estimation

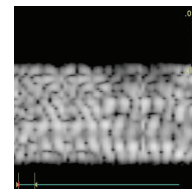
Clutter filtering effect on the speckle images

- Axial and lateral object movement is equal to a rotation of the system point spread function around the k_z - ω and k_x - ω axes in k -space
- Ideal clutter filter removes all signal below a threshold ω -plane
- The remaining signal may become *band-pass* in both k_z and k_x

K-space representation:



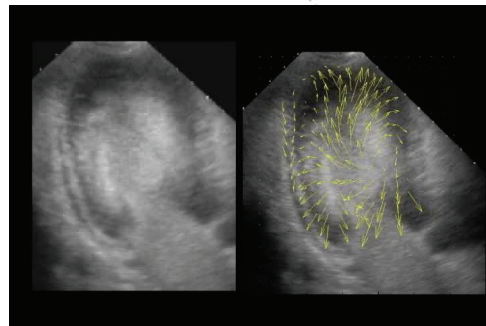
FIR filtered, ~60 deg



FIR filtered, ~90 deg

Echo particle image velocimetry (Echo-PIV)

- Tracking the movement of small concentrations of injected contrast bubbles
- Avoids problems with clutter filtering and low SNR
- Can be used to estimate flow-derived parameters such as wall shear rate



Example: Left ventricle flow patterns by Echo-PIV

Reference: G.-R. Hong et al. *Characterization and Quantification of Vortex Flow in the Human Left Ventricle by Contrast Echocardiography Using Vector Particle Image Velocimetry*, JACC: Cardiovascular Imaging, Volume 1, Issue 6, November 2008

Summary – 2-D velocity estimation

- Compound vector-Doppler and lateral oscillation methods give basically the same performance
- Both vector-Doppler and speckle tracking are able to resolve the axial *and* lateral velocity component, however:
 - a) Clutter filtering will cause signal drop-outs for both methods
 - b) Vector-Doppler accuracy is dependent on the angle between tx and rx apertures, limiting the depth and width of the scan possible
 - c) The receive apertures are limited in vector-Doppler (single array operation) leading to a reduction in signal-to-noise ratio
 - d) Speckle tracking is dependent on a very high frame rate due to speckle decorrelation, especially for complex flow patterns
 - e) Speckle tracking is dependent on spatial interpolation in order to achieve accurate displacement estimates

High frame rate CFI

Increasing the frame rate in CFI

Generate several receive lines per transmit

- Parallel receive beamforming
- Plane-wave flow imaging
- Synthetic aperture flow imaging

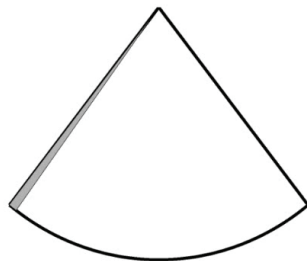
Multiple transmit pulses

- Disjoint in frequency, filtering on receive to separate

Decrease the ensemble size

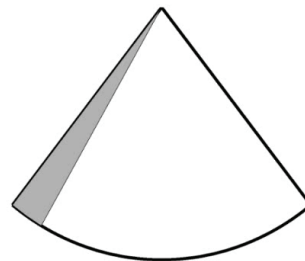
- More advanced clutter filtering is needed

Parallel receive beamforming



Conventional image formation:

An image is built line by line, one for each ultrasound transmission

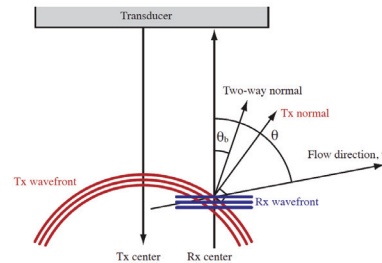
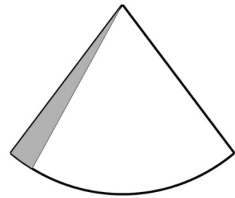


Parallell receive beamforming:

N image lines are generated per ultrasound transmission
Frame rate increased by N times

Find out more at the poster presentation by T. Hergum in the Beamforming session at this years conference

Parallel receive beamforming

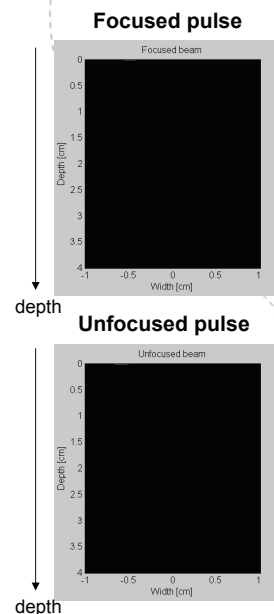


- + N image lines are generated per ultrasound transmission, i.e. **frame rate increases by N** . Based on traditional focused beams on transmit.
- The **curvature of focused transmit beams** leads to a Doppler angle varying across the group of parallel receive beams
- For a large number of PRB the **transmit aperture becomes very small**

Reference: T. Hergum et al., *Reducing Color Flow Artifacts caused by Parallel Beamforming*, IEEE Trans. Ultrason., Ferrielect. And Freq. Contr. submitted, 2009

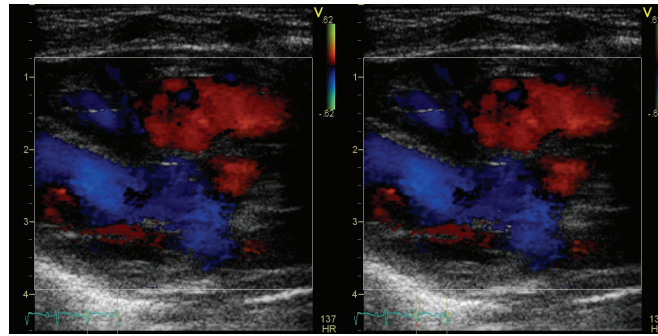
Plane-wave imaging

- Plane-waves are **unfocused ultrasound beams, without curvature** over a limited depth
- One transmitted plane-wave can illuminate a large spatial region so that **a high number of parallel receive beams can be utilized**
- Due to the flat curvature, **image artifacts** normally associated with parallel receive beamforming for focused beams **are avoided**
- **Trade-off:** A significant loss in transmitted pressure compared to focused beams



What is the difference?

An illustration of the frame rate difference for the same amount of image lines (fully sampled). Playback is at 20% of real-time.



Equivalent to current high-end acquisition, **2x parallel beams**

Plane wave imaging, **16x parallel beams** (both B-mode and CFI)

Image example:

Neonate heart with an atrium-ventricular septum defect. Parasternal cross-section.

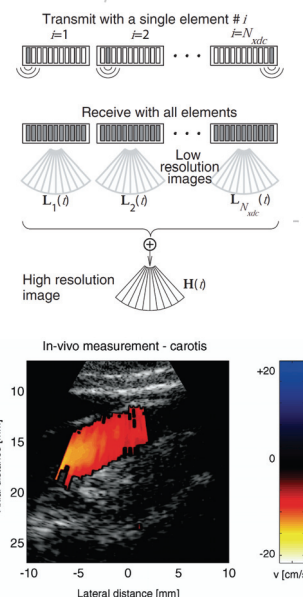
More examples are shown at this years conference, Wednesday 23. Sept at the oral Blood Flow session

Synthetic aperture flow imaging

- One emission results in a complete image with low resolution and SNR
- By combining several low-resolution images a high-resolution image is produced

Pros and cons

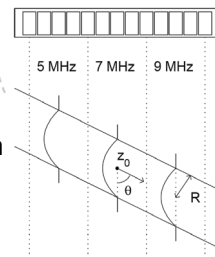
- + Continuous acquisition
- + Flexible beamforming
- SNR issues
- Computational demands
- Motion artifacts due to inadequate compensation of movement for high-resolution images



Reference: S. I. Nikolov and J. A. Jensen, *In-Vivo Synthetic Aperture Flow Imaging in Medical Ultrasound*, IEEE Trans. Ultrason., Ferroelect. And Freq. Contr. vol. 50, no. 7, July 2008

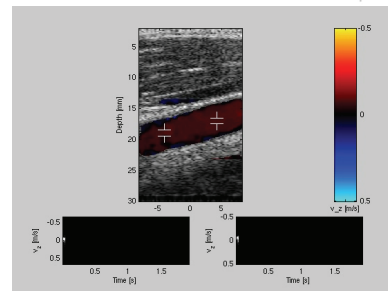
Multiple transmit pulses

- Emit N wide-band transmit pulses simultaneously in different directions
- Filter signals from disjoint (narrow-band) frequency bands on receive



Pros and cons

- + Increases the frame rate by N
- + Conventional focused beams
- + May be combined with parallel receive beamforming
- Varying estimator quality for different frequency bands
- Varying Nyquist velocity for each band
- Varying SNR for each band



Reference: N. Odderhede et al, *Multi-Frequency Encoding for Fast Color Flow or Quadroplex Imaging*, IEEE Trans. Ultrason., Ferrielect. And Freq. Contr. vol. 55, no. 4, April 2008

Real-time 3-D imaging

15
9

3D color flow imaging with 2-D matrix array probe

From 1D Array...



64 – 128 array elements

to 2D Array...



2000 – 3000 array elements

Challenges:

Long data acquisition time, stitching data from 7 heart cycles

Small element size → high electrical impedance → lower SNR



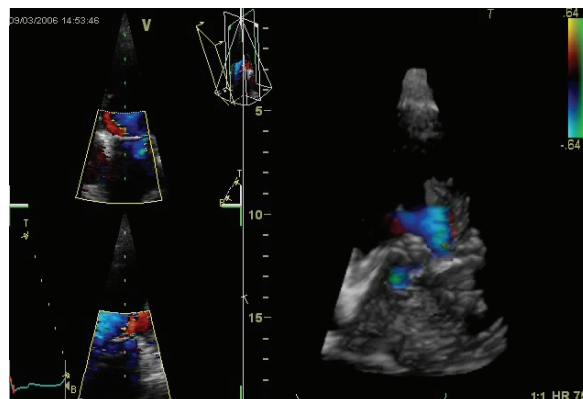
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Real-time 3-D CFI example



Example:

Mitral valve insufficiency

Real-time CFI without ECG triggering and volume stitching

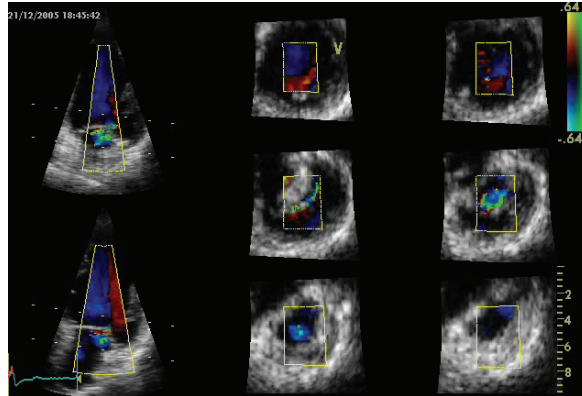
The main challenge is frame rate

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Real-time 3-D CFI example



Example:

Mitral valve insufficiency

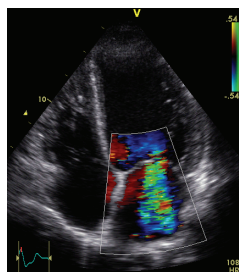
Real-time CFI without ECG triggering and volume stitching

6-slice view – notice the banana shaped leakage in the mitral valve indicated by the green color (increased Doppler bandwidth)

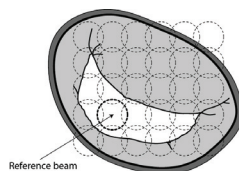
New information!

Estimation of Valvular regurgitation Area by 3D HPRF Doppler

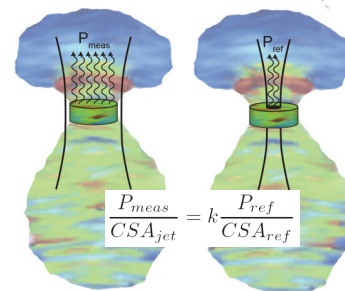
Valvular regurgitant jet area can be quantified by isolating the Doppler signal power from the jet over the whole leakage area relative to a known reference signal from the center of the jet (vena contracta)



Mitral valve insufficiency

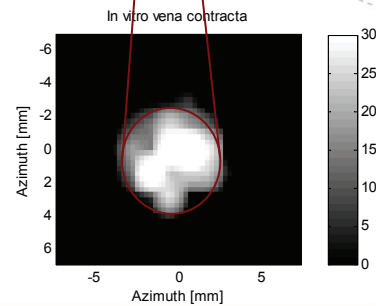
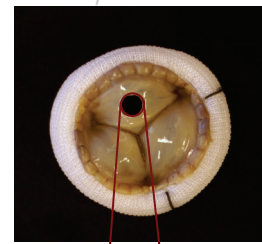
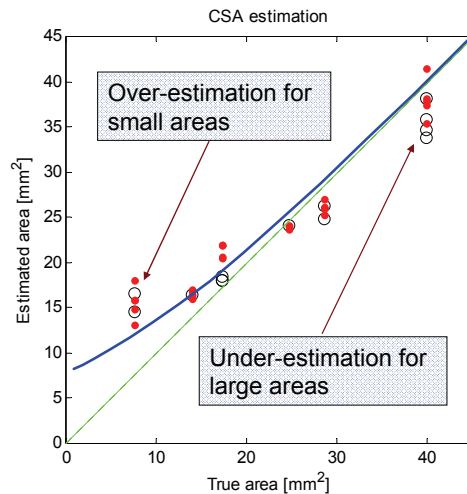


Mitral valve 3-D beam pattern



16
3

In vitro results



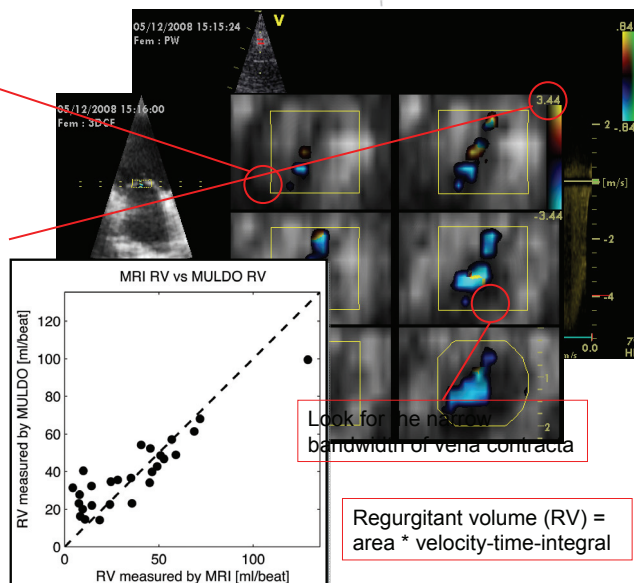
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4

In vivo - compared to MRI

- Vena contracta position determined with PW Doppler.
- Jet power isolated with high pulse repetition frequency (HPRF).
- Acquire 3D HPRF data.
- Obtains jet geometry, cross sectional area, and regurgitant volume.
- Small leakages overestimated when compared to MRI. This is expected due to the finite beam width.



www.ntnu.no

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Thanks for your attention!

(We hope you enjoyed the show)

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A. Støylen (NTNU), A. Heimdel (GE Vingmed), S. Bjærum (GE Vingmed), K. Kristoffersen (GE Vingmed), T. Hegrum (NTNU), J. Crosby (NTNU), F. Orderud (NTNU), J. Udesen (DTU), J. U. Voigt, S. A. Aase (NTNU), A. Swillens (UGent), P. Segers (UGent), and more...

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